

# COMPLETE ANALYSIS ON THE NLO SUSY-QCD CORRECTIONS TO $B^0 - \bar{B}^0$ MIXING

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## Abstract

We present a complete next-to-leading-order calculation of the QCD corrections to  $B^0 - \bar{B}^0$  ( $K^0 - \bar{K}^0$ ) mixing in the framework of the minimal flavor violating (MFV) supersymmetry. We take into account the contributions from the gluino and find that the gluino-mediated corrections modify the LO result obviously even when the mass of gluino  $m_{\tilde{g}} \gg m_w$ . In general, one cannot neglect gluino contributions.

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## 1 Introduction

As the most popular candidate for new physics beyond the standard model (SM), the supersymmetry[1] has been studied extensively during the last two decades. Even so, there is no experimental evidence for any of the new particles predicted by various supersymmetry (SUSY) models at present. Before new colliders are available searching for new physics, we should focus on indirect probes of the phenomena induced by SUSY at low energies. At this point, the most promising processes that we can depend on are the Flavor Changing

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Neutral Current (FCNC) processes, especially  $b \rightarrow s\gamma$  and oscillations of neutral mesons. For weak decays at presence of the strong interaction, evaluating such processes requires special technique. The main tool to calculate concerned quantities of those processes is the effective Hamiltonian theory. It is a two step program, starting with an operator product expansion (OPE)[2, 3] and performing a renormalization group equation (RGE)[4] analysis afterwards. The necessary machinery has been developed over years[5, 6, 7, 8, 9]. The new physics effects on the rare B processes are discussed in literature. The calculation of the rate of inclusive decay  $B \rightarrow X_s\gamma$  is presented by authors of [10, 11, 12] in the two-Higgs doublet model (THDM). The supersymmetric effect on  $B \rightarrow X_s\gamma$  is discussed in [13, 14, 15] and the NLO QCD corrections are given in [16]. The transition  $b \rightarrow s\gamma\gamma$  in the supersymmetric extension of standard model is computed in [17]. The hadronic B decays[18] and CP-violation in those processes[19] have been discussed also. The authors of [20] have discussed possibility of observing supersymmetric effects in the rare decays  $B \rightarrow X_s\gamma$  and  $B \rightarrow X_se^+e^-$  in the B-factory. Studies on decays  $B \rightarrow (K, K^*)l^+l^-$  in SM and supersymmetric model have been carried out in [21]. The SUSY effects on these processes are very interesting and studies on them may shed some light on the general characteristics of the SUSY model. A relevant review can be found in [22]. For oscillations of  $B_0 - \bar{B}_0$  ( $K_0 - \bar{K}_0$ ), calculations have been done in the Standard Model (SM) and THDM. As for the supersymmetric extension of SM, the calculation involving the gluino contributions should be re-studied carefully for gluino has non-zero mass. In this paper, we will present a complete analysis of SUSY-QCD corrections to the oscillations of  $B_0 - \bar{B}_0$  ( $K_0 - \bar{K}_0$ ) in the supersymmetric extension of SM with the minimal flavor violation, i.e. the flavor violation occurs only via the charged current at the tree level.

Our main results can be summarized as follows:

- We give a complete computation of supersymmetric QCD-corrections to  $B_0 - \bar{B}_0$  oscillations up to NLO, our technique can be applied to compute SUSY-QCD corrections to other rare B-decay processes (such as  $b \rightarrow s\gamma$  etc.) up to NLO.
- Additionally, we find that the gluino contribution to the NLO-QCD corrections grows as  $\ln x_{\tilde{g}w}$  when gluino is heavier than the lightest up-type scalar quark, where  $x_{\tilde{g}w} = \frac{m_{\tilde{g}}^2}{m_w^2}$ .

At the next-to-leading order approximation, the QCD corrections to the  $B^0 - \bar{B}^0$  mixing in the SUSY model have been discussed recently. The authors of [23, 24] applied the mass-insertion method to estimate QCD correction effects on the  $B^0 - \bar{B}^0$  mixing. However, in their work, the contribution from gluino ( $\tilde{g}$ ) was ignored. It was thought that at high energies,  $\alpha_s/4\pi \ll 1$ , so that the correction induced by  $\tilde{g}$  might be

discarded. The authors of [25] noticed the significance of the gluino contributions, however they only gave a general discussion without carrying out any concrete calculation on the gluino contributions. The calculation including the gluino-mediated QCD corrections needs new technique for  $m_{\tilde{g}} > m_t$ . In this work, our method is analogous to that employed in [25], but we develop the technique and handle all problems carefully, then draw our conclusion about the size of the gluino contributions through a reliable calculation.

The paper is organized as follows. In section 2, we display the necessary parts of the MSSM-Feynman rules and give the effective Hamiltonian without QCD-corrections. In section 3, we discuss the features of the NLO calculation with special focus on the explicit QCD- corrections and matching-procedure. Furthermore, we show that reasonably removing the contributions from SUSY-particles and the physical charged Higgs  $H^\pm$ , our result turns back to the SM result[26]. In section 4 we give the numerical results of the NLO and scan the extent of the parameter space in the MSSM with minimal flavor violation. We close this paper with conclusions and discussions. Some technical details are collected in the long appendices.

## 2 Notation and the box-diagram results

### 2.1 Notation and the Feynman-Rules

Throughout this paper we adopt the notation of [27], the expressions of the concerned propagators and vertices can be found in the Appendix of [27]. For convenience, we give the superpotential and relevant mixing matrices. The most general form of the superpotential which does not violate gauge invariance and the conservation laws in SM is

$$\mathcal{W} = \mu\epsilon_{ij}\hat{H}_i^1\hat{H}_j^2 + \epsilon_{ij}h_l^{IJ}\hat{H}_i^1\hat{L}_j^I\hat{R}^J + \epsilon_{ij}h_d^{IJ}\hat{H}_i^1\hat{Q}_j^I\hat{D}^J + \epsilon_{ij}h_u^{IJ}\hat{H}_i^2\hat{Q}_j^I\hat{U}^J. \quad (1)$$

Here  $\hat{H}^1, \hat{H}^2$  are Higgs superfields;  $\hat{Q}^I$  and  $\hat{L}^I$  are quark and lepton superfields in doublets of the weak SU(2) group, where I=1, 2, 3 are the indices of generations; the rest superfields:  $\hat{U}^I$  and  $\hat{D}^I$  being quark superfields of u- and d-types, and  $\hat{R}^I$  charged leptons are in singlets of the weak SU(2). The indices i, j are contracted in a general way for the SU(2) group, and  $h_l, h_{u,d}$  are the Yukawa couplings. Taking into account of the soft breaking terms, we can study the phenomenology within the minimal supersymmetric extension of the standard model (MSSM). One difference between the MSSM and SM is the Higgs sector. There are four charged scalars, two of them are physical massive Higgs bosons and other are massless Goldstones. The mixing matrix can be written as:

$$\mathcal{Z}_H = \begin{pmatrix} \sin \beta & -\cos \beta \\ \cos \beta & \sin \beta \end{pmatrix} \quad (2)$$

with  $\tan \beta = \frac{v_2}{v_1}$  and  $v_1, v_2$  are the vacuum-expectation values of the two Higgs scalars. Another matrix that we will use is the chargino mixing matrix. The SUSY partners of charged Higgs and  $W^\pm$  combine to give four Dirac fermions:  $\kappa_1^\pm, \kappa_2^\pm$ . The two mixing matrices  $\mathcal{Z}^\pm$  appearing in the Lagrangian are defined as

$$(\mathcal{Z}^-)^T \mathcal{M}_c \mathcal{Z}^+ = \text{diag}(m_{\kappa_1}, m_{\kappa_2}), \quad (3)$$

where  $\mathcal{M}_c$  is the mass matrix of charginos. In a similar way,  $Z_{U,D}$  diagonalize the mass matrices of the up- and down-type squarks respectively:

$$\mathcal{Z}_{U,D}^\dagger \mathcal{M}_{\tilde{q}}^2 \mathcal{Z}_{U,D} = \text{diag}(m_{\tilde{U}, \tilde{D}}^2). \quad (4)$$

We present the relevant vertices in Fig.1 and Fig.2.  $a, b, c$  are the indices of SU(3) group in appropriate representations. We have explicitly written down the Yukawa-type couplings for the up-type quarks. For the down-type quarks, we use the symbol  $h_{dI} = \frac{m_{dI}}{\cos \beta m_w}$  to represent the Yukawa couplings and the short-hand notation  $\omega_\pm = \frac{1 \pm \gamma_5}{2}$  for the left- and right-handed projectors.

## 2.2 Box-diagram results

At absence of QCD corrections, the effective Hamiltonian for the  $B^0 - \bar{B}^0$  mixing is obtained by evaluating the box diagrams (Fig.3). Neglecting external momenta and masses, the effective Hamiltonian for  $\Delta B = 2$  transitions at the weak-scale is[28]

$$H_{eff}^0 = \frac{G_F^2}{4\pi^2} m_w^2 \sum_{ij} \sum_{\alpha} \lambda_i \lambda_j S_{\alpha} \mathcal{O}_{\alpha} \quad (5)$$

where  $\lambda_i = V_{ib} V_{id}^*$  ( $V_{ij}$  are the elements of the CKM matrix with  $i, j = 1, 2, 3$ ) and the operators  $\mathcal{O}_{\alpha}$  are defined as

$$\mathcal{O}_1 = \bar{d} \gamma_{\mu} \omega_{-} b \bar{d} \gamma^{\mu} \omega_{-} b,$$

$$\mathcal{O}_2 = \bar{d} \gamma_{\mu} \omega_{-} b \bar{d} \gamma^{\mu} \omega_{+} b,$$

$$\mathcal{O}_3 = \bar{d} \omega_{-} b \bar{d} \omega_{+} b,$$

$$\mathcal{O}_4 = \bar{d} \omega_{-} b \bar{d} \omega_{-} b,$$

$$\mathcal{O}_5 = \bar{d} \sigma_{\mu\nu} \omega_{-} b \bar{d} \sigma^{\mu\nu} \omega_{-} b,$$

$$\mathcal{O}_6 = \bar{d} \gamma_{\mu} \omega_{+} b \bar{d} \gamma^{\mu} \omega_{+} b,$$

$$\mathcal{O}_7 = \bar{d}\omega_+ b \bar{d}\omega_+ b,$$

$$\mathcal{O}_8 = \bar{d}\sigma_{\mu\nu}\omega_+ b \bar{d}\sigma^{\mu\nu}\omega_+ b, \quad (6)$$

with the parameter  $x_{i\text{w}} = \frac{m_i^2}{m_{\text{w}}^2}$ , the coefficients  $S_\alpha$  are given as

$$\begin{aligned} S_1 &= \left( f_a(x_{i\text{w}}, x_{j\text{w}}, 1, 1) - 2 \frac{\mathcal{Z}_H^{2k} \mathcal{Z}_H^{2k*}}{\sin^2 \beta} x_{i\text{w}} x_{j\text{w}} f_b(x_{i\text{w}}, x_{j\text{w}}, 1, x_{H_k^- \text{w}}) \right. \\ &\quad \left. + \frac{x_{i\text{w}} x_{j\text{w}}}{4 \sin^4 \beta} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2k*} \mathcal{Z}_H^{2l} \mathcal{Z}_H^{2l*} f_a(x_{i\text{w}}, x_{j\text{w}}, x_{H_k^- \text{w}}, x_{H_l^- \text{w}}) \right. \\ &\quad \left. - \frac{1}{4} a_{+,i}^{km} b_{-,j}^{kn} a_{+,j}^{ln} b_{-,i}^{lm} f_a(x_{\tilde{U}^i_{m\text{w}}}, x_{\tilde{U}^j_{n\text{w}}}, x_{\kappa_k^- \text{w}}, x_{\kappa_l^- \text{w}}) \right), \\ S_2 &= \frac{1}{4} \left( \frac{h_d h_d}{2 \sin^2 \beta} \left( \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k*} \mathcal{Z}_H^{2l} \mathcal{Z}_H^{1l*} x_{j\text{w}} + \mathcal{Z}_H^{2k} \mathcal{Z}_H^{1k*} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2l*} x_{i\text{w}} \right) f_a(x_{i\text{w}}, x_{j\text{w}}, x_{H_k^- \text{w}}, x_{H_l^- \text{w}}) \right. \\ &\quad \left. + \left( a_{+,i}^{km} b_{+,j}^{kn} a_{-,j}^{ln} b_{-,i}^{lm} + a_{-,i}^{km} b_{-,j}^{kn} a_{+,j}^{ln} b_{+,i}^{lm} \right) \sqrt{x_{\kappa_k \text{w}} x_{\kappa_l \text{w}}} f_b(x_{\tilde{U}^i_{m\text{w}}}, x_{\tilde{U}^j_{n\text{w}}}, x_{\kappa_k^- \text{w}}, x_{\kappa_l^- \text{w}}) \right) \\ &\quad - \frac{1}{4} \left( -2 h_d h_b \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1k*} f_a(x_{i\text{w}}, x_{j\text{w}}, 1, x_{H_k^- \text{w}}) \right. \\ &\quad \left. + \frac{x_{i\text{w}} x_{j\text{w}}}{\sin^2 \beta} h_d h_b \left( \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2k*} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{1l*} + \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1k*} \mathcal{Z}_H^{2l} \mathcal{Z}_H^{2l*} \right) f_b(x_{i\text{w}}, x_{j\text{w}}, x_{H_k^- \text{w}}, x_{H_l^- \text{w}}) \right. \\ &\quad \left. + \frac{1}{2} \left( a_{+,i}^{km} b_{-,j}^{kn} a_{-,j}^{ln} b_{+,i}^{lm} + a_{-,i}^{km} b_{+,j}^{kn} a_{+,j}^{ln} b_{-,i}^{lm} \right) f_a(x_{\tilde{U}^i_{m\text{w}}}, x_{\tilde{U}^j_{n\text{w}}}, x_{\kappa_k^- \text{w}}, x_{\kappa_l^- \text{w}}) \right), \\ S_3 &= -2S_2, \\ S_4 &= \frac{1}{4} \left( \frac{x_{i\text{w}} x_{j\text{w}}}{\sin^2 \beta} h_d^2 \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k*} \mathcal{Z}_H^{2l*} \mathcal{Z}_H^{1l} f_b(x_{i\text{w}}, x_{j\text{w}}, x_{H_k^- \text{w}}, x_{H_l^- \text{w}}) \right. \\ &\quad \left. + \frac{1}{2} a_{-,i}^{km} b_{-,j}^{kn} a_{-,j}^{ln} b_{-,i}^{lm} f_b(x_{\tilde{U}^i_{m\text{w}}}, x_{\tilde{U}^j_{n\text{w}}}, x_{\kappa_k^- \text{w}}, x_{\kappa_l^- \text{w}}) \right) \\ &\quad - \frac{3}{8} a_{-,i}^{km} b_{-,j}^{kn} a_{-,j}^{ln} b_{-,i}^{lm} f_b(x_{\tilde{U}^i_{m\text{w}}}, x_{\tilde{U}^j_{n\text{w}}}, x_{\kappa_k^- \text{w}}, x_{\kappa_l^- \text{w}}), \\ S_5 &= \frac{1}{4} S_4, \\ S_6 &= \left( \frac{1}{4} h_d^2 h_b^2 \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1k*} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{1l*} f_a(x_{i\text{w}}, x_{j\text{w}}, x_{H_k^- \text{w}}, x_{H_l^- \text{w}}) \right. \\ &\quad \left. - \frac{1}{4} a_{-,i}^{km} b_{+,j}^{kn} a_{-,j}^{ln} b_{+,i}^{lm} f_a(x_{\tilde{U}^i_{m\text{w}}}, x_{\tilde{U}^j_{n\text{w}}}, x_{\kappa_k^- \text{w}}, x_{\kappa_l^- \text{w}}) \right), \\ S_7 &= \frac{1}{4} \left( \frac{x_{i\text{w}} x_{j\text{w}}}{\sin^2 \beta} h_b^2 \mathcal{Z}_H^{2k} \mathcal{Z}_H^{1k*} \mathcal{Z}_H^{1l*} \mathcal{Z}_H^{2l} f_b(x_{i\text{w}}, x_{j\text{w}}, x_{H_k^- \text{w}}, x_{H_l^- \text{w}}) \right. \\ &\quad \left. + \frac{1}{2} a_{+,i}^{km} b_{+,j}^{kn} a_{+,j}^{ln} b_{+,i}^{lm} f_b(x_{\tilde{U}^i_{m\text{w}}}, x_{\tilde{U}^j_{n\text{w}}}, x_{\kappa_k^- \text{w}}, x_{\kappa_l^- \text{w}}) \right) \\ &\quad - \frac{3}{8} a_{+,i}^{km} b_{+,j}^{kn} a_{+,j}^{ln} b_{+,i}^{lm} f_b(x_{\tilde{U}^i_{m\text{w}}}, x_{\tilde{U}^j_{n\text{w}}}, x_{\kappa_k^- \text{w}}, x_{\kappa_l^- \text{w}}), \end{aligned}$$

$$S_8 = \frac{1}{4}S_7. \quad (7)$$

The functions  $f_{a,b}(x_1, x_2, x_3, x_4)$  are given in the appendix.A and the new symbols  $a_{\pm}, b_{\pm}$  are defined as

$$\begin{aligned} a_{+,i}^{j,k} &= -Z_{\tilde{U}^i}^{1j} Z_{1k}^{+*} + \frac{x_{iw}}{\sqrt{2} \sin \beta} Z_{\tilde{U}^i}^{2j} Z_{2k}^{+*}, \\ a_{-,i}^{j,k} &= h_d Z_{\tilde{U}^i}^{1j} Z_{2k}^-, \\ b_{+,i}^{j,k} &= h_b Z_{\tilde{U}^i}^{1j} Z_{2k}^{+*}, \\ b_{-,i}^{j,k} &= -Z_{\tilde{U}^i}^{1j*} Z_{1k}^+ + \frac{x_{iw}}{\sqrt{2} \sin \beta} Z_{\tilde{U}^i}^{2j*} Z_{2k}^+. \end{aligned} \quad (8)$$

On purpose, we keep the Yukawa-couplings of the down-type quarks explicitly in Eq.5, so that we can discuss any possible value of  $\tan \beta$  in the Higgs sector. This is different from some early works[29, 25]. Another point which should be emphasized is that Eq.5 can recover the one-loop result of [25] as long as considering the unitarity of the CKM matrix and discarding the Yukawa couplings of down-type quarks.

### 3 Explicit QCD corrections to the box diagram

#### 3.1 The general method to compute the two-loop integral

In this section, we will give the explicit perturbative QCD correction up to  $\mathcal{O}(\alpha_s)$ . The Feynman diagrams are drawn in Fig.4, Fig.5 and Fig.6. Similar to the previous treatments[26, 29, 30], we will carry out the calculation in an arbitrary covariant  $\xi$ -gauge for the gluon propagator, where  $\xi = 0$  represents the Feynman-'t Hooft gauge and  $\xi = 1$  the Landau gauge. The W-propagators is set in the Feynman-'t Hooft gauge.

The two-loop Feynman diagrams including all SUSY particles can be categorized into five distinct topological classes (a),(b),(c),(d) and (e) in Fig.7. Fig.7(c) and Fig.7(d) are the self energy- and vertex-insertion diagrams respectively, whereas the other three classes are of complicated topological structures.

Fig.4(a, c, g), Fig.5(a, c, g) and Fig.6 (a, b, c, d) belong to the topological class shown in Fig.7(a); Fig.4(b) and Fig.5(b) belong to the topological class in Fig.7(b); Fig.4(f) and Fig.5(f) belong to the topological class in Fig.7(e); Fig.4(d), Fig.5(d) and Fig.6(e, f) belong to the topological class in Fig.7(c); Fig.4(e), Fig.5(e) and Fig.6(g, h, i, j) belong to the topological class in Fig.7(d). The double penguin diagrams Fig.4(h) and Fig.5(h) do not contribute for vanishing external momenta.

To obtain the physical quantities, we have to deal with ultraviolet divergence. The divergence stems from diagrams Fig.4(d, e), Fig.5(d, e) and Fig.6(e, f, g, h, i, j). In this case we employ dimensional regularization [31, 33] and we carry out the renormalization in the  $\overline{\text{MS}}$ -scheme[31, 32].

For an effective Hamiltonian, all internal particles must be integrated out, namely, a condition that there exists at least one internal particle with  $m_{int} \gg m_{ext}$  where  $m_{int}$  and  $m_{ext}$  refer to the masses of the internal and all external particles (bosons or fermions) respectively, is implied. In the case for  $B^0 - \bar{B}^0$  or  $K^0 - \bar{K}^0$  mixing,  $m_b$  and  $m_d$  should be set as zero in the resultant effective theory. At the 0-th order, e.g. when calculating the box diagrams, there is no problem in the limit of  $m_b \sim m_d = 0$ . However, when the QCD corrections are taken into account,  $b$ - and  $d$ -quark lines become internal in diagrams Fig.4(a, b, c) and Fig.5(a, b, c), then under the limit  $m_b \sim m_d = 0$  an infrared divergence emerges. The divergence is artificial and can be eliminated in the full theory. The natural way to handle this problem is keeping all internal-line masses to be non-zero at denominator of the propagators.

As well known, we need to achieve the effective Hamiltonian at lower energies and the QCD-corrected box diagrams would determine the boundary condition of RGE for the running of the Wilson coefficients. Therefore, the infrared divergence must be properly eliminated. In next section, following the standard procedures given in literature to build a matching between the full theory and the effective one at the scale  $\mu$ , we can get rid of the troublesome infrared divergence.

In the THDM sector of MSSM, the calculation is standard and consistent with the previous work, but because of the large masses of gluino and squarks, we need to take a more general treatment for calculating the contribution of the two-loop diagrams which include gluino. In the following part, we will illustrate how to compute the loop integral and separate ultraviolet divergence in one example. In (d) of Fig.7, We have an integral as:

$$I_{(d),2}^a = \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{(2\pi)^D} \frac{k^4}{(k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_3^2)(k^2 - m_4^2)((k+q)^2 - m_5^2)(q^2 - m_6^2)(q^2 - m_7^2)} \quad (9)$$

where the  $m_i$   $i = 1, \dots, 7$  are the internal line (bosons or fermions) masses. The above integral can be decomposed as

$$I_{(d),2}^a = I_{(d),2}^{a,1} + (m_3^2 + m_4^2)I_{(d),1}^a - m_3^2 m_4^2 I_{(d),0}, \quad (10)$$

with

$$\begin{aligned} I_{(d),2}^{a,1} &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{(2\pi)^D} \frac{1}{(k^2 - m_1^2)(k^2 - m_2^2)((k+q)^2 - m_5^2)(q^2 - m_6^2)(q^2 - m_7^2)}, \\ I_{(d),1}^a &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{(2\pi)^D} \frac{k^2}{(k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_3^2)(k^2 - m_4^2)((k+q)^2 - m_5^2)(q^2 - m_6^2)(q^2 - m_7^2)}, \\ I_{(d),0} &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{(2\pi)^D} \frac{1}{(k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_3^2)(k^2 - m_4^2)((k+q)^2 - m_5^2)(q^2 - m_6^2)(q^2 - m_7^2)}. \end{aligned}$$

(11)

Now, we calculate the loop integral  $I_{D,2}^{a,1}$  step by step. After the Wick rotation, it is written as:

$$\begin{aligned}
I_{(d),2}^{a,1} &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{(2\pi)^D} \frac{1}{(k^2 + m_1^2)(k^2 + m_2^2)((k+q)^2 + m_5^2)(q^2 + m_6^2)(q^2 + m_7^2)} \\
&= \frac{1}{(m_1^2 - m_2^2)(m_6^2 - m_7^2)} \int \frac{d^D k}{(2\pi)^D} \left( \frac{m_1^2}{k^2(k^2 + m_1^2)} - \frac{m_2^2}{k^2(k^2 + m_2^2)} \right) \\
&\quad \frac{1}{(k+q)^2 + m_5^2} \left( \frac{1}{q^2 + m_6^2} - \frac{1}{q^2 + m_7^2} \right) \\
&= \frac{1}{(m_1^2 - m_2^2)(m_6^2 - m_7^2)} \frac{B(\frac{D}{2}, \varepsilon) B(2\varepsilon, 1 - \varepsilon)}{\Gamma^2(\frac{D}{2})(4\pi)^D} \left( \frac{m_1^2}{m_1^{4\varepsilon}} \int dx x^{-\varepsilon} (1-x)^\varepsilon \right. \\
&\quad \left. \left( F(\varepsilon, 2\varepsilon; 1 + \varepsilon; 1 - \frac{x_{51}}{x} - \frac{x_{61}}{1-x}) - F(\varepsilon, 2\varepsilon; 1 + \varepsilon; 1 - \frac{x_{51}}{x} - \frac{x_{71}}{1-x}) \right) - (m_1^2 \rightarrow m_2^2) \right) \quad (12)
\end{aligned}$$

with  $x_{ij} = \frac{m_i^2}{m_j^2}$ .  $F(\alpha, \beta; \gamma; t)$  is the hypergeometric function [34] and  $\varepsilon = 2 - \frac{D}{2}$ . To derive Eq.12, we employ formula[35]

$$\int_0^\infty dt t^{\lambda-1} (1+t)^{-\mu+\nu} (t+\beta)^{-\nu} = B(\mu - \lambda, \lambda) F(\nu, \mu - \lambda; \mu; 1 - \beta) \quad (13)$$

with  $Re\mu > Re\lambda > 0$ . Using the definition of the hypergeometric function[34, 35]

$$\begin{aligned}
F(\alpha, \beta; \gamma; z) &= 1 + \frac{\alpha \cdot \beta}{\gamma \cdot 1} z + \frac{\alpha(\alpha+1)\beta(\beta+1)}{\gamma(\gamma+1) \cdot 1 \cdot 2} z^2 \\
&\quad + \frac{\alpha(\alpha+1)(\alpha+2)\beta(\beta+1)(\beta+2)}{\gamma(\gamma+1)(\gamma+2) \cdot 1 \cdot 2 \cdot 3} z^3 + \dots, \quad (14)
\end{aligned}$$

we have

$$\begin{aligned}
F(\varepsilon, 2\varepsilon; 1 + \varepsilon; z) &= 1 + \frac{2\varepsilon^2}{1 \cdot 1} z + \frac{2\varepsilon^2 \cdot 1 \cdot 1}{(1 \cdot 2)(1 \cdot 2)} z^2 + \frac{2\varepsilon^2(1 \cdot 2)(1 \cdot 2)}{(1 \cdot 2 \cdot 3)(1 \cdot 2 \cdot 3)} z^3 \\
&\quad + \dots + \frac{2\varepsilon^2(n-1)!(n-1)!}{n!n!} z^n + \dots \\
&= 1 + 2\varepsilon^2 \int_0^2 dz \left( 1 + \frac{1 \cdot 1}{2 \cdot 1} z + \frac{(1 \cdot 2)(1 \cdot 2)}{2 \cdot 3 \cdot 2!} z^2 + \dots + \frac{(n-1)!(n-1)!}{n!(n-1)!} z^{n-1} + \dots \right) + \dots \\
&= 1 + 2\varepsilon^2 \int_0^z dz F(1, 1; 2; z) + \dots \\
&= 1 + 2\varepsilon^2 L_{i_2}(z) + \dots \quad (15)
\end{aligned}$$



This is the key formula to proceed our computation and  $L_{i_2}(z)$  is the dilogarithm function, which is defined as

$$L_{i_2}(z) = - \int_0^z dt \frac{\ln(1-t)}{t} = \sum_{n=1}^{\infty} \frac{z^n}{n^2}, \quad |z| < 1. \quad (16)$$

Using Eq.15, we have

$$I_{(d),2}^{a,1} = \frac{1}{4\pi^4} \frac{1}{(m_1^2 - m_2^2)(m_6^2 - m_7^2)} \left( m_1^2 \left( \mathcal{S}L_{i_2}(x_{51}, x_{61}) - \mathcal{S}L_{i_2}(x_{51}, x_{71}) \right) \right. \\ \left. - m_2^2 \left( \mathcal{S}L_{i_2}(x_{51}, x_{61}) - \mathcal{S}L_{i_2}(x_{51}, x_{71}) \right) \right) \quad (17)$$

and  $\mathcal{S}L_{i_2}(a, b)$  is

$$\mathcal{S}L_{i_2}(a, b) = \int_0^1 dt L_{i_2} \left( 1 - \frac{a}{t} - \frac{b}{1-t} \right),$$

which is a continuous and analytic function[36], whose general expression can be found in Appendix.C. In the above example,  $I_{(d),2}^{a,1}$  does not contain divergence. Now, let us look at another part that contains ultraviolet divergence. In the same diagram Fig.7(d), there exists  $I_{(d),2}^d$  which is ultraviolet divergent, the corresponding integral is

$$I_{(d),2}^d = \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{(2\pi)^D} \frac{k^2 q^2}{(k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_3^2)(k^2 - m_4^2)((k+q)^2 - m_5^2)(q^2 - m_6^2)(q^2 - m_7^2)} \\ = I_{(d),2}^{d,1} + m_7^2 I_{(d),1}^a + m_4^2 I_{(d),1}^b - m_4^2 m_7^2 I_{(d),0}. \quad (18)$$

The explicit forms of  $I_{(d),2}^{d,1}$ ,  $I_{(d),1}^{a,1}$ ,  $I_{(d),1}^{b,1}$ ,  $I_{(d),2}^{c,1}$ ,  $I_{(d),0}$ , are given in Appendix.B. For convenience, we calculate only one of them as an example to display how to deal with them and obtain corresponding result. In the above expression, the form of  $I_{(d),2}^{d,1}$  is

$$I_{(d),2}^{d,1} = \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{(2\pi)^D} \frac{1}{(k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_3^2)((k+q)^2 - m_5^2)(q^2 - m_6^2)}. \quad (19)$$

Explicitly, we have a solution

$$I_{(d),2}^{d,1} = \frac{1}{(4\pi)^4} \sum_{i=1}^3 \frac{m_i^2}{\prod_{j \neq i} (m_j^2 - m_i^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln 4\pi \right) \ln x_{i\text{w}} + \left( 3 - \gamma_E + \ln 4\pi \right) \ln x_{i\text{w}} \right. \\ \left. - \ln^2 x_{i\text{w}} - \mathcal{S}L_{i_2}(x_{5i}, x_{6i}) \right). \quad (20)$$

Generally, in the self-energy (class Fig.7(c)) and vertex (class Fig.7(d)) insertion diagrams, there is ultraviolet divergence which needs to be renormalized; in the other topological classes (Fig.7(a, b, e)), no ultraviolet

divergence exists. Certain renormalization procedures can eliminate the ultraviolet divergence, here we employ the  $\overline{\text{MS}}$  (the modified minimal subtraction scheme) to do the job.

Now let us turn to possible infrared divergence which may occur in the integrations.

We expand the two-loop results with respect to  $m_b, m_d$  to order  $\mathcal{O}(m_{b,d})$  and then let the masses of the down-type quarks (d, s and b) approach to zero. Explicitly,  $\mathcal{S}L_{i_2}(a, b)$  is written as

$$\begin{aligned} \mathcal{S}L_{i_2}(a, b) = & \left( 3 - \frac{\pi^2}{6} - \ln a \ln(1-a) + a \ln a \ln \frac{a-1}{a} - aL_{i_2}\left(\frac{1}{a}\right) - L_{i_2}(a) \right) \\ & + \frac{b}{a-1} \left( a \left( \frac{-\pi^2}{6} + \ln(a(1-a)) \ln \frac{a-1}{a} + L_{i_2}\left(\frac{a}{a-1}\right) - L_{i_2}\left(\frac{1}{a}\right) + L_{i_2}\left(\frac{1}{1-a}\right) \right) \right. \\ & \left. - \left( \ln a \ln b - L_{i_2}(a) - \frac{1}{2} \ln^2 a - \ln(a(1-a)) \right) \right) + \mathcal{O}(b^2) \end{aligned} \quad (21)$$

with  $b \rightarrow 0$ . Obviously, Eq.(21) indicates that as  $m_b \sim m_d = 0$ ,  $I_{(a),2}^{a,b,c,d,e,f}$ ,  $I_{(a),1}^{a,b,c}$  and  $I_{(b),2}^{a,b,c,d,e,f}$ ,  $I_{(b),1}^{a,b,c}$  would blow up, however the superficial infrared divergence is benign as long as we retain the masses of the down-type quarks to be non-zero.

As discussed above, the QCD correction to the effective Hamiltonian of Eq.5 is given as follows

$$\Delta H_{eff} = \frac{G_F^2}{4\pi^2} m_w^2 \frac{\alpha_s}{4\pi} \sum_{i,j} \lambda_i \lambda_j U_{i,j}, \quad (22)$$

where

$$U_{i,j} = \sum_k^8 \phi_k \mathcal{O}_k, \quad (23)$$

with  $\mathcal{O}_k$  being defined in Eq.6 and  $\phi_k$  are written as

$$\phi_k = \phi_k^g + \phi_k^{\tilde{g}}. \quad (24)$$

$\phi_k^g$  (k=1, 2, ..., 8) are the contributions of gluon and  $\phi_k^{\tilde{g}}$  come from gluino corrections.  $\phi_k^g$  have been derived and are of following forms

$$\begin{aligned} \phi_1^g = & L_{i,j}^1 - \xi \left( \frac{10}{3} S_1 + \frac{1}{6} S_4 + \frac{1}{6} S_7 \right) - \frac{10}{3} (1-\xi) \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} S_1 \\ & + \left( \frac{4}{3} - \frac{1}{3} \xi \right) \ln x_{dw} x_{bw} S_1 + \frac{8}{3} (1-\xi) \ln x_\mu S_1 - (4-\xi) S_2 \frac{m_b m_d}{2(m_d^2 - m_b^2)} \ln \frac{x_{dw}}{x_{bw}} \\ & + 2 \ln x_\mu (\nabla_x S_1), \\ \phi_2^g = & L_{i,j}^2 - \frac{11}{3} \xi S_2 + \left( -\frac{25}{3} + \frac{10}{3} \xi \right) S_2 \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} + \frac{1}{2} (2-\xi) S_2 \ln x_{bw} x_{dw} \end{aligned}$$

$$\begin{aligned}
& + (4 - \xi) \left( \frac{5}{6} S_1 - \frac{5}{12} S_4 + \frac{5}{6} S_6 - \frac{5}{12} S_7 \right) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}} \\
& + \frac{8}{3} (1 - \xi) \ln x_\mu S_2 + 2 \ln x_\mu (\nabla_x S_2), \\
\phi_3^g &= L_{i,j}^3 + \frac{22}{3} \xi S_2 + \left( \frac{50}{3} - \frac{20}{3} \xi \right) \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} + (2 - \xi) S_2 \ln x_{dw} x_{bw} \\
& - (4 - \xi) \left( \frac{5}{3} S_1 - \frac{5}{6} S_4 + \frac{5}{3} S_6 - \frac{5}{6} S_7 \right) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}} \\
& - \frac{16}{3} (1 - \xi) \ln x_\mu S_2 - 4 \ln x_\mu (\nabla_x S_2), \\
\phi_4^g &= L_{i,j}^4 + \xi \left( \frac{1}{3} S_1 - \frac{10}{3} S_4 - \frac{1}{3} S_6 \right) + \left( -\frac{4}{3} + \frac{10}{3} \xi \right) S_4 \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} \\
& + \frac{2 - \xi}{3} S_4 \ln x_{dw} x_{bw} + \frac{5}{6} (4 - \xi) S_2 \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}} \\
& + \frac{8}{3} (1 - \xi) \ln x_\mu S_4 + 2 \ln x_\mu (\nabla_x S_4), \\
\phi_5^g &= L_{i,j}^5 + \xi \left( \frac{1}{12} S_1 - \frac{5}{6} S_4 - \frac{1}{12} S_6 \right) + \left( -\frac{1}{3} + \frac{5}{6} \xi \right) S_4 \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} \\
& + \frac{2 - \xi}{12} S_4 \ln x_{dw} x_{bw} + \frac{5}{24} (4 - \xi) S_2 \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}} \\
& + \frac{2}{3} (1 - \xi) \ln x_\mu S_4 + \frac{1}{2} \ln x_\mu (\nabla_x S_4), \\
\phi_6^g &= L_{i,j}^6 - \xi \left( \frac{10}{3} S_6 + \frac{1}{6} S_4 + \frac{1}{6} S_7 \right) - \frac{10}{3} (1 - \xi) \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} S_6 \\
& + \left( \frac{4}{3} - \frac{1}{3} \xi \right) \ln x_{dw} x_{bw} S_6 + \frac{8}{3} (1 - \xi) \ln x_\mu S_6 - (4 - \xi) S_2 \frac{m_b m_d}{2(m_d^2 - m_b^2)} \ln \frac{x_{dw}}{x_{bw}} \\
& + 2 \ln x_\mu (\nabla_x S_6), \\
\phi_7^g &= L_{i,j}^7 + \xi \left( \frac{1}{3} S_6 - \frac{10}{3} S_7 - \frac{1}{3} S_1 \right) + \left( -\frac{4}{3} + \frac{10}{3} \xi \right) S_7 \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} \\
& + \frac{2 - \xi}{3} S_7 \ln x_{dw} x_{bw} + \frac{5}{6} (4 - \xi) S_2 \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}} \\
& + \frac{8}{3} (1 - \xi) \ln x_\mu S_7 + 2 \ln x_\mu (\nabla_x S_7), \\
\phi_8^g &= L_{i,j}^8 + \xi \left( \frac{1}{12} S_6 - \frac{5}{6} S_7 - \frac{1}{12} S_1 \right) + \left( -\frac{1}{3} + \frac{5}{6} \xi \right) S_7 \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} \\
& + \frac{2 - \xi}{12} S_7 \ln x_{dw} x_{bw} + \frac{5}{24} (4 - \xi) S_2 \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}} \\
& + \frac{2}{3} (1 - \xi) \ln x_\mu S_7 + \frac{1}{2} \ln x_\mu (\nabla_x S_7), \tag{25}
\end{aligned}$$

where  $x_\mu = \frac{\mu^2}{m_w^2}$  and  $\mu$  is the scale at which the heavy degrees of freedom are integrated out.

It is noted that there are  $\log x_i$  terms in the expressions. In fact, the situation for the Feynman diagrams including the vertex and self-energy insertions is more subtle, because these kinds of diagrams are logarithmically divergent. When the masses of the inner loop (vertex loop or self-energy) are much greater than that of the outer loop, <sup>2</sup> logarithmic divergence  $\log \frac{m_i^2}{m_e^2}$  may emerge where  $m_i$  is the mass of the particles in the inner loop (vertex correction or self-energy) and  $m_e$  is the mass of particles in the outer loop [45].

Here, we have defined a new symbol

$$\begin{aligned} \nabla_x = & 3x_{iw} \frac{\partial}{\partial x_{iw}} + 3x_{jw} \frac{\partial}{\partial x_{jw}} 2x_{H_k^- w} \frac{\partial}{\partial x_{H_k^- w}} + 2x_{H_l^- w} \frac{\partial}{\partial x_{H_l^- w}} + 3x_{\tilde{\kappa}_k^- w} \frac{\partial}{\partial x_{\tilde{\kappa}_k^- w}} \\ & + 3x_{\tilde{\kappa}_l^- w} \frac{\partial}{\partial x_{\tilde{\kappa}_l^- w}} + 2x_{\tilde{U}^i_{mw}} \frac{\partial}{\partial x_{\tilde{U}^i_{mw}}} + 2x_{\tilde{U}^i_{nw}} \frac{\partial}{\partial x_{\tilde{U}^i_{nw}}}. \end{aligned} \quad (26)$$

One should note that all the masses entering the functions are the masses evaluated at the  $\mu$  scale.  $L_{i,j}^a$  ( $a = 1, \dots, 8$ ) are complicated functions of inner line masses, which are collected in the appendix. When we derive the above results, the Fierz-transformation is used to organize the emerging operators into the form of color-singlet current  $\otimes$  color-singlet current. As for the diagrams contain the ultraviolet divergence, we have taken the  $\mathcal{O}(\varepsilon)$  contributions into account seriously. The results depend on the gauge parameter, and are infrared-divergent as  $m_{b,d} \rightarrow 0$ . However, the infrared divergence and gauge-dependence vanish after we match the full and effective sides of the theory and the explicit procedure of the matching is shown in next subsection.

### 3.2 Wilson coefficient function of $\mathcal{O}_i$

The effective Hamiltonian to order  $\mathcal{O}(\alpha_s)$  is given as

$$H_{eff} = H_{eff}^0 + \Delta H_{eff}, \quad (27)$$

where  $H_{eff}^0$  is the pure box contribution and  $\Delta H_{eff}$  is the Hamiltonian resulted in by the SUSY-QCD corrections. To obtain the Wilson coefficients in Eq.(27), one needs to properly handle the matching condition between the full theory and effective one.

As stated above, the Hamiltonian contains the infrared divergence and gauge dependence. In order to obtain physics results, we need to match the effective theory to the full theory. Before doing this, we evaluate

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<sup>2</sup> Here "inner loop" refers to the loop for the inserted self-energy and vertex corrections, whereas "outer loop" is for the loop part outside the "inner loop". These notations are taken to distinguish "inner" and "outer" from "internal" and "external" quantities in the loop evaluation to avoid possible ambiguities.

the matrix elements of the physical operators  $\mathcal{O}_i (i = 1, \dots, 8)$  up to order  $O(\alpha_s)$  using the same regularization, renormalization and gauge prescriptions employed above. The one-loop diagrams which are responsible for the corrections to the operators  $\mathcal{O}_i$  are given in Fig.8, the results are

$$\mathcal{O}_i = \mathcal{O}_i^{(0)} + \frac{\alpha_s}{4\pi^2} \sum_j \left( C_F r_{ij}^{(1)} \mathcal{O}_j^{(1)} + T^a \otimes T^a r_{ij}^{(8)} \tilde{\mathcal{O}}_j^{(8)} \right) \quad (28)$$

with

$$\begin{aligned} r_{11}^{(1)} &= -3 + 2(1 - \xi) \left( 1 + \ln x_\mu - \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} \right), \\ r_{12}^{(1)} &= (4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}}, \\ r_{11}^{(8)} &= -5 - (4 - \xi)(2 \ln x_\mu - \ln x_{dw} x_{bw}) + 2(1 - \xi) \left( 1 + \ln x_\mu - \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} \right), \\ r_{13}^{(8)} &= -2(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}}, \\ r_{14}^{(8)} &= r_{17}^{(8)} = -(4 - \xi); \\ r_{15}^{(8)} &= r_{18}^{(8)} = \frac{1}{4}(4 - \xi); \\ r_{21}^{(1)} &= \frac{1}{2}(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}}, \\ r_{22}^{(1)} &= -3 + 2(1 - \xi) \left( 1 + \ln x_\mu - \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} \right), \\ r_{26}^{(1)} &= \frac{1}{2}(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}}, \\ r_{22}^{(8)} &= (4 - \xi) \left( \ln x_{dw} x_{bw} - 2 \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} \right) - 8 - 2\xi, \\ r_{23}^{(8)} &= -2(4 - \xi), \\ r_{24}^{(8)} &= r_{27}^{(8)} = -(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}}, \\ r_{25}^{(8)} &= r_{28}^{(8)} = \frac{4 - \xi}{4} \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}}, \\ r_{33}^{(1)} &= 4 - 2\xi + 2(4 - \xi) \left( \ln x_\mu - \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} \right), \\ r_{34}^{(1)} &= r_{37}^{(1)} = -(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}}, \\ r_{31}^{(8)} &= \frac{1}{4}(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}}, \end{aligned}$$

$$\begin{aligned}
r_{32}^{(8)} &= -\frac{1}{2}(4 - \xi), \\
r_{33}^{(8)} &= -\frac{5}{2} - 2\xi + 2(1 - \xi) \left( \ln x_{d\text{w}} x_{b\text{w}} - \frac{x_{d\text{w}} \ln x_{d\text{w}} - x_{b\text{w}} \ln x_{b\text{w}}}{x_{d\text{w}} - x_{b\text{w}}} \right), \\
r_{36}^{(8)} &= \frac{1}{4}(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{d\text{w}}}{x_{b\text{w}}}, \\
r_{43}^{(1)} &= -2(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{d\text{w}}}{x_{b\text{w}}}, \\
r_{44}^{(1)} &= 4 - 2\xi + 2(4 - \xi) \left( \ln x_\mu - \frac{x_{d\text{w}} \ln x_{d\text{w}} - x_{b\text{w}} \ln x_{b\text{w}}}{x_{d\text{w}} - x_{b\text{w}}} \right), \\
r_{41}^{(8)} &= -\frac{1}{4}(4 - \xi), \\
r_{42}^{(8)} &= -\frac{1}{2}(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{d\text{w}}}{x_{b\text{w}}}, \\
r_{44}^{(8)} &= (1 - \xi) \left( 2 + \ln x_{d\text{w}} x_{b\text{w}} - 2 \frac{x_{d\text{w}} \ln x_{d\text{w}} - x_{b\text{w}} \ln x_{b\text{w}}}{x_{d\text{w}} - x_{b\text{w}}} \right), \\
r_{45}^{(8)} &= \frac{3}{4} - \ln x_\mu + \frac{1}{4} \ln x_{b\text{w}} x_{d\text{w}} + \frac{1}{2} \frac{x_{d\text{w}} \ln x_{d\text{w}} - x_{b\text{w}} \ln x_{b\text{w}}}{x_{d\text{w}} - x_{b\text{w}}}, \\
r_{46}^{(8)} &= -\frac{1}{4}(4 - \xi), \\
r_{55}^{(1)} &= 2\xi \left( \frac{x_{d\text{w}} \ln x_{d\text{w}} - x_{b\text{w}} \ln x_{b\text{w}}}{x_{d\text{w}} - x_{b\text{w}}} - \ln x_\mu - 1 \right), \\
r_{51}^{(8)} &= 3(4 - \xi), \\
r_{52}^{(8)} &= -24(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{d\text{w}}}{x_{b\text{w}}}, \\
r_{54}^{(8)} &= -32 + 48 \ln x_\mu - 12 \ln x_{d\text{w}} x_{b\text{w}} - 24 \frac{x_{d\text{w}} \ln x_{d\text{w}} - x_{b\text{w}} \ln x_{b\text{w}}}{x_{d\text{w}} - x_{b\text{w}}}, \\
r_{55}^{(8)} &= 2(3 + \xi) \frac{x_{d\text{w}} \ln x_{d\text{w}} - x_{b\text{w}} \ln x_{b\text{w}}}{x_{d\text{w}} - x_{b\text{w}}} - (3 + \xi) \ln x_{d\text{w}} x_{b\text{w}} - 2(1 + \xi), \\
r_{66}^{(1)} &= -3 + 2(1 - \xi) \left( 1 + \ln x_\mu - \frac{x_{d\text{w}} \ln x_{d\text{w}} - x_{b\text{w}} \ln x_{b\text{w}}}{x_{d\text{w}} - x_{b\text{w}}} \right), \\
r_{62}^{(1)} &= (4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{d\text{w}}}{x_{b\text{w}}}, \\
r_{66}^{(8)} &= -5 - (4 - \xi)(2 \ln x_\mu - \ln x_{d\text{w}} x_{b\text{w}}) + 2(1 - \xi) \left( 1 + \ln x_\mu - \frac{x_{d\text{w}} \ln x_{d\text{w}} - x_{b\text{w}} \ln x_{b\text{w}}}{x_{d\text{w}} - x_{b\text{w}}} \right), \\
r_{63}^{(8)} &= -2(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{d\text{w}}}{x_{b\text{w}}}, \\
r_{64}^{(8)} &= r_{67}^{(8)} = -(4 - \xi);
\end{aligned}$$

$$\begin{aligned}
r_{65}^{(8)} &= r_{68}^{(8)} = \frac{1}{4}(4 - \xi), \\
r_{73}^{(1)} &= -2(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}}, \\
r_{77}^{(1)} &= 4 - 2\xi + 2(4 - \xi) \left( \ln x_\mu - \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} \right), \\
r_{76}^{(8)} &= -\frac{1}{4}(4 - \xi), \\
r_{72}^{(8)} &= -\frac{1}{2}(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}}, \\
r_{77}^{(8)} &= (1 - \xi) \left( 2 + \ln x_{dw} x_{bw} - 2 \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} \right), \\
r_{78}^{(8)} &= \frac{3}{4} - \ln x_\mu + \frac{1}{4} \ln x_{bw} x_{dw} + \frac{1}{2} \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}}, \\
r_{71}^{(8)} &= -\frac{1}{4}(4 - \xi), \\
r_{88}^{(1)} &= 2\xi \left( \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} - \ln x_\mu - 1 \right), \\
r_{86}^{(8)} &= 3(4 - \xi), \\
r_{82}^{(8)} &= -24(4 - \xi) \frac{m_b m_d}{m_d^2 - m_b^2} \ln \frac{x_{dw}}{x_{bw}}, \\
r_{87}^{(8)} &= -32 + 48 \ln x_\mu - 12 \ln x_{dw} x_{bw} - 24 \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}}, \\
r_{88}^{(8)} &= 2(3 + \xi) \frac{x_{dw} \ln x_{dw} - x_{bw} \ln x_{bw}}{x_{dw} - x_{bw}} - (3 + \xi) \ln x_{dw} x_{bw} - 2(1 + \xi). \tag{29}
\end{aligned}$$

The other elements of  $r^{(1,8)}$  are zero. At the scale  $\mu$  where matching between the full Hamiltonian and the effective one is made, the matching condition can be written as

$$\begin{aligned}
H_{eff} &= H_{eff}^0 + \Delta H_{eff} \\
&\equiv H_{full} = \frac{G_F^2}{4\pi^2} \lambda_i \lambda_j \left( \vec{\mathcal{O}}^{(0)T} \cdot \left[ \vec{S} + \frac{\alpha_s}{4\pi} \vec{\phi} \right] \right) \\
&= \frac{G_F^2}{4\pi^2} \lambda_i \lambda_j \vec{\mathcal{O}}^T(\mu) \cdot \vec{C}(\mu) \tag{30}
\end{aligned}$$

where  $\vec{\mathcal{O}}^{(0)}$  are the tree-level operators, but  $\vec{\mathcal{O}}(\mu)$  are the QCD-modified operators and  $\vec{C}(\mu)$  are the corresponding coefficients. From the Eq.28 and Eq.29, we obtain

$$\vec{\mathcal{O}}(\mu) = \left( 1 + \frac{\alpha_s}{4\pi} \hat{r} \right) \vec{\mathcal{O}}^{(0)}, \tag{31}$$

where matrix  $\hat{r}$  can be obtained from  $r^{(1)}$ ,  $r^{(8)}$  and is read as

$$\hat{r} = \frac{1}{2} \left[ C_F r^{(1)} + \frac{1}{2} r^{(8)} \cdot \mathcal{F} - \frac{1}{6} r^{(8)} \right] \cdot (\hat{I} + \mathcal{F}), \quad (32)$$

where  $\hat{I}$  denotes the unit matrix and  $\mathcal{F}$  is the Fierz transformation matrix in the basis Eq.6:

$$\mathcal{F} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{8} \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & \frac{1}{2} \end{pmatrix} \quad (33)$$

The coefficients  $\vec{C}(\mu)$  are obtained by comparing Eq.30 with Eq.31[37]:

$$\vec{C}(\mu) = \vec{S} + \frac{\alpha_s}{4\pi} (\vec{\phi} - \hat{r}^T \vec{S}) \quad (34)$$

where  $\vec{C}(\mu)$  are given as

$$\begin{aligned} C_1(\mu) &= S_1 + \frac{\alpha_s}{4\pi} \left[ \phi_{i,j}^{\tilde{g},1} + L_{i,j}^1 + C_F (S_1 + 2 \ln x_\mu S_1 + 2 \ln x_\mu \nabla_x S_1) \right. \\ &\quad \left. + C_A ((3 + 6 \ln x_\mu) S_1 + 4(S_4 + S_5) - (S_7 + S_8)) \right], \\ C_2(\mu) &= S_2 + \frac{\alpha_s}{4\pi} \left[ \phi_{i,j}^{\tilde{g},2} + L_{i,j}^2 + C_F \left( -\frac{1}{2} S_2 - 4 \ln x_\mu S_2 + 2 \ln x_\mu \nabla_x S_2 \right) \right. \\ &\quad \left. + \frac{13}{12} S_2 \right], \\ C_3(\mu) &= S_3 + \frac{\alpha_s}{4\pi} \left[ \phi_{i,j}^{\tilde{g},3} + L_{i,j}^3 + C_F (S_2 + 8 \ln x_\mu S_2 + 2 \ln x_\mu \nabla_x S_3) \right. \\ &\quad \left. - \frac{13}{6} S_2 \right], \\ C_4(\mu) &= S_4 + \frac{\alpha_s}{4\pi} \left[ \phi_{i,j}^{\tilde{g},4} + L_{i,j}^4 + C_F (-12 S_4 - 24 \ln x_\mu S_4 + 2 \ln x_\mu \nabla_x S_4) \right. \\ &\quad \left. - \frac{143}{12} S_1 + \frac{191}{6} S_4 + \frac{31}{16} S_5 + \frac{1}{12} S_6 - 24 \ln x_\mu S_4 + \frac{1}{12} \ln x_\mu S_5 \right], \\ C_5(\mu) &= S_5 + \frac{\alpha_s}{4\pi} \left[ \phi_{i,j}^{\tilde{g},5} + L_{i,j}^5 + C_F (-12 S_5 - 24 \ln x_\mu S_5 + 2 \ln x_\mu \nabla_x S_5) \right. \\ &\quad \left. - \frac{143}{48} S_1 + \frac{191}{24} S_4 + \frac{31}{64} S_5 + \frac{1}{48} S_6 - 6 \ln x_\mu S_4 + \frac{1}{48} \ln x_\mu S_5 \right], \\ C_6(\mu) &= S_6 + \frac{\alpha_s}{4\pi} \left[ \phi_{i,j}^{\tilde{g},6} + L_{i,j}^6 + C_F (S_6 + 2 \ln x_\mu S_6 + 2 \ln x_\mu \nabla_x S_6) \right. \\ &\quad \left. + C_A ((3 + 6 \ln x_\mu) S_6 + 4(S_7 + S_8) - (S_4 + S_5)) \right], \end{aligned}$$



$$\begin{aligned}
C_7(\mu) &= S_7 + \frac{\alpha_s}{4\pi} \left[ \phi_{i,j}^{\tilde{g},7} + L_{i,j}^7 + C_F \left( -12S_7 - 24 \ln x_\mu S_7 + 2 \ln x_\mu \nabla_x S_7 \right) \right. \\
&\quad \left. - \frac{143}{12} S_6 + \frac{191}{6} S_7 + \frac{31}{16} S_8 + \frac{1}{12} S_1 - 24 \ln x_\mu S_7 + \frac{1}{12} \ln x_\mu S_8 \right], \\
C_8(\mu) &= S_8 + \frac{\alpha_s}{4\pi} \left[ \phi_{i,j}^{\tilde{g},8} + L_{i,j}^8 + C_F \left( -12S_8 - 24 \ln x_\mu S_8 + 2 \ln x_\mu \nabla_x S_8 \right) \right. \\
&\quad \left. - \frac{143}{48} S_6 + \frac{191}{24} S_7 + \frac{31}{64} S_8 + \frac{1}{48} S_1 - 6 \ln x_\mu S_7 + \frac{1}{48} \ln x_\mu S_8 \right],
\end{aligned} \tag{35}$$

with  $C_F = \frac{4}{3}$ ,  $C_A = \frac{1}{3}$ , and  $S_1$  through  $S_8$  are defined in Eq.(7). Hence, at this stage we have the expressions for the Wilson- coefficients at the matching scale  $\mu$ , which do not suffer from the infrared divergence under limit of  $m_b \sim m_d = 0$ , and this is consistent with the requirements for the effective Hamiltonian. The next step is to perform an evolution down to lower scales. The renormalization group equation for the Wilson coefficients  $\vec{C}$  reads

$$\left[ \mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} - \frac{\hat{\gamma}^T}{2} \right] \vec{C}(\mu, \alpha_s) = 0 \tag{36}$$

where  $\hat{\gamma}$  is the anomalous-dimension matrix and  $\beta(\alpha_s)$  is the usual  $\beta$  function. The solution of the Eq.36 is discussed in [37] and we only cite the result here. Through the renormalization-group evolution matrix  $\hat{W}(m, \mu)$ , the vectors  $\vec{C}(m)$  can be written as

$$\vec{C}(m) = \hat{W}(m, \mu) \vec{C}(\mu) \tag{37}$$

with

$$\hat{W}(m, \mu) = \left( 1 + \frac{\alpha_s(m)}{4\pi} \hat{J}(m) \right) U(\hat{m}, \mu) \left( 1 + \frac{\alpha_s(\mu)}{4\pi} \hat{J}(\mu) \right)^{-1}, \tag{38}$$

where  $\hat{U}$  is the leading-order evolution matrix

$$\hat{U}(m, \mu) = \left[ \frac{\alpha_s(\mu)}{\alpha_s(m)} \right]^{\hat{\gamma}^{0T}/2\beta_0} \tag{39}$$

and the matrix  $\hat{J}$  is given in [37]. To obtain the above formulae, the computer algebra system MATHEMATICA 4.0[42] and MATHEMATICA-based package FeynArts[43] are used. The package TRACER[44] is used to evaluate the spinor structure.

The main purpose of this work is investigating the gluino corrections to the  $B^0 - \overline{B}^0$  mixing in the supersymmetric scenario with minimal flavor violation. Before proceeding our discussion, we would analyze the gluino corrections to  $\Delta H_{eff}$  first. In order to understand the point thoroughly, we neglect the mixing between the right- and left- squarks. In the case,  $\mathcal{Z}_{\tilde{Q}^i}^{12} = \mathcal{Z}_{\tilde{Q}^i}^{21} = 0$ ,  $\mathcal{Z}_{\tilde{Q}^i}^{11} = \mathcal{Z}_{\tilde{Q}^i}^{22} = 1$  and  $m_{\tilde{Q}_1^i} = m_{\tilde{Q}_R^i}$ ,

$m_{\tilde{Q}_2^i} = m_{\tilde{Q}_L^i}$ . In the computation of corrections from gluinos to  $B^0 - \bar{B}^0$  mixing, the following terms will appear in the coefficients of  $Q_1(\mu)$  (Fig.6(g,h))

$$C_1(\mu) \propto -i \frac{G_F^2}{4\pi^2} m_w^2 \frac{\alpha_s}{4\pi} \lambda_t \lambda_t^* \left\{ 2C_F \sum_{\alpha\beta} \mathcal{Z}_{\tilde{D}^3}^{1\alpha} \mathcal{Z}_{\tilde{D}^3}^{1\alpha*} \mathcal{Z}_{\tilde{U}^3}^{1\beta} \mathcal{Z}_{\tilde{U}^3}^{1\beta*} \left[ F_D^{2e} - F_D^{2a} - F_D^{2d} \right] (x_1, x_2, x_3, x_4, x_{\tilde{D}_\alpha^3}, x_{\tilde{g}_w}, x_{\tilde{U}_\beta^3}) \right\}, \quad (40)$$

where  $x_i$  ( $i = 1, 2$ ) represent  $x_{u_{Iw}}$  ( $I=1, 2, 3$ ) and  $x_k$  ( $k = 3, 4$ ) represent  $x_{H_l^- w}$  ( $l = 1, 2$ ). When  $x_{\tilde{g}} \gg x_{\tilde{D}_\alpha^3}, x_{\tilde{U}_\beta^3}$ , we have

$$\begin{aligned} C_1(\mu) &\propto i \frac{G_F^2}{4\pi^2} m_w^2 \frac{\alpha_s}{4\pi} \lambda_t \lambda_t^* \left[ 2C_F \sum_{i=1}^4 \frac{x_{iw}^2 \ln x_{iw}}{\prod_{j \neq i} (x_{jw} - x_{iw})} \right] \ln x_{\tilde{g}_w} \\ &= i \frac{G_F^2}{4\pi^2} m_w^2 \frac{\alpha_s}{4\pi} \lambda_t \lambda_t^* \left[ \frac{2x_{tw} \ln x_{tw}}{(x_{tw} - 1)^3} - \frac{1 + x_{tw}}{(x_{tw} - 1)^2} \right] \ln x_{\tilde{g}_w} \end{aligned} \quad (41)$$

Here, we have presumed  $\tan \beta \sim 1$  and the contributions to other  $C_i(\mu)$  ( $i = 2, \dots, 8$ ) are suppressed by the small Yukawa couplings  $h_b, h_d$ . In Eq.41, we have set  $x_1 = x_2 = x_{tw}$  and  $x_3 = x_4 = 1$  (this choice corresponds to exchanging  $W$ -boson and top quark in the outer loop). Similar analysis can be performed in calculating the contributions of Fig.5(e,f) (self-insertion diagrams), and we will find the amplitude growing with  $\ln x_{\tilde{g}_w}$  when  $m_{\tilde{g}} \gg m_{\tilde{U}_1^3}$ . A similar conclusion is derived in the SM, where the one-loop radiation corrections to mass of the  $W$ -boson is increasing with  $\ln m_h$  ( $m_h$  is the mass of the standard Higgs)[41]. When  $\tan \beta \gg 1$ , the corrections to the coefficients  $C_i(\mu)$  ( $i = 2, \dots, 8$ ) must be taken into account seriously, because those terms cannot cancel each other among themselves and are enhanced strongly when the mass of gluino  $m_{\tilde{g}}$  is much greater than  $m_{\tilde{U}_1^3}$ . If we consider the mixing between the left- and right-squarks, the expressions would be very complicated and we present them in the appendix. We will further discuss the gluino corrections in the section of numerical results. However, for illustration of the physics picture, neglecting such mixing would not bring up any confusion.

### 3.3 Hadronic Matrix Elements

To numerically evaluate the  $B^0 - \bar{B}^0$  ( $K^0 - \bar{K}^0$ ) mixing, besides the low-energy effective  $\Delta B = 2$  Lagrangian, one needs to properly calculate the hadronic matrix elements of the various operators in Eq.27. Estimation of such hadronic matrix elements is notoriously difficult, and is generally accompanied by large uncertainties due to long-distance, non-perturbative QCD effects. Fortunately, although the same case holds in the current

context, there are two factors which mitigate those hadronic uncertainties in our ensuring phenomenological analysis:

- The supersymmetric contributions to  $\overline{B}^0 - B^0$  and  $\overline{K}^0 - K^0$  mixing in the MSSM with minimal flavor violation give rise to the same operator  $\mathcal{O}_1$  that exists in the standard model. This makes comparison of the supersymmetry and standard model contributions relatively straightforward.
- For the  $\overline{B}^0 - B^0$  system, the vacuum saturation approximation employed below is believed to be a good approximation. This belief is supported by the lattice Monte Carlo estimates which give  $B_B \simeq 1$ [38, 39].

We begin by restating the conventional result for the operator  $\mathcal{O}_1$ :

$$\langle K^0 | \mathcal{O}_1 | \overline{K}^0 \rangle = \frac{1}{3} f_K^2 m_K^2 B_K^1, \quad (42)$$

where  $f_K \simeq 165 \text{ MeV}$  is the K-meson decay constant and  $B_K^1 = 1$  corresponds to the "vacuum saturation" result. Various estimates of this matrix element place  $B_K^1$  in the range of  $0.3 \sim 1$ [40], with a value  $B_K^1 \sim 0.7$  is favored by the lattice gauge results[38, 39]. Matrix elements of the other hadronic operators  $\mathcal{O}_i$  ( $i = 2, \dots, 8$ ) can be written as

$$\begin{aligned} \langle K^0 | \mathcal{O}_2 | \overline{K}^0 \rangle &= - \left[ \frac{1}{4} - \frac{1}{6} \left( \frac{m_K}{m_s + m_d} \right)^2 \right] m_K^2 f_K^2 B_K^2, \\ \langle K^0 | \mathcal{O}_3 | \overline{K}^0 \rangle &= \left[ \frac{1}{24} - \frac{1}{4} \left( \frac{m_K}{m_s + m_d} \right)^2 \right] m_K^2 f_K^2 B_K^3, \\ \langle K^0 | \mathcal{O}_4 | \overline{K}^0 \rangle &= \frac{5}{24} \left( \frac{m_K}{m_s + m_d} \right)^2 m_K^2 f_K^2 B_K^4, \\ \langle K^0 | \mathcal{O}_5 | \overline{K}^0 \rangle &= \frac{1}{4} \left( \frac{m_K}{m_s + m_d} \right)^2 m_K^2 f_K^2 B_K^5, \\ \langle K^0 | \mathcal{O}_6 | \overline{K}^0 \rangle &= \frac{1}{3} f_K^2 m_K^2 B_K^6, \\ \langle K^0 | \mathcal{O}_7 | \overline{K}^0 \rangle &= \frac{5}{24} \left( \frac{m_K}{m_s + m_d} \right)^2 m_K^2 f_K^2 B_K^7, \\ \langle K^0 | \mathcal{O}_8 | \overline{K}^0 \rangle &= \frac{1}{4} \left( \frac{m_K}{m_s + m_d} \right)^2 m_K^2 f_K^2 B_K^8. \end{aligned} \quad (43)$$

Similarly, the factors  $B_K^i$  ( $i = 2, \dots, 8$ ) are associated with each of the matrix elements in Eq.43.

The corresponding results for the  $\Delta B = 2$  matrix elements are simplified by the fact that the current algebra enhancement factor  $\frac{m_B}{m_b + m_d} \simeq 1$  is sufficiently accurate to present experimental tolerance. Thus we

have

$$\begin{aligned}
\langle B^0 | \mathcal{O}_1 | \overline{B}^0 \rangle &= \frac{1}{3} f_B^2 m_B^2, \\
\langle B^0 | \mathcal{O}_2 | \overline{B}^0 \rangle &= -\frac{1}{12} f_B^2 m_B^2, \\
\langle B^0 | \mathcal{O}_3 | \overline{B}^0 \rangle &= -\frac{5}{24} f_B^2 m_B^2, \\
\langle B^0 | \mathcal{O}_4 | \overline{B}^0 \rangle &= \frac{5}{24} f_B^2 m_B^2, \\
\langle B^0 | \mathcal{O}_5 | \overline{B}^0 \rangle &= \frac{1}{4} f_B^2 m_B^2, \\
\langle B^0 | \mathcal{O}_6 | \overline{B}^0 \rangle &= \frac{1}{3} f_B^2 m_B^2, \\
\langle B^0 | \mathcal{O}_7 | \overline{B}^0 \rangle &= \frac{5}{24} f_B^2 m_B^2, \\
\langle B^0 | \mathcal{O}_8 | \overline{B}^0 \rangle &= \frac{1}{4} f_B^2 m_B^2,
\end{aligned} \tag{44}$$

However, the potential benefits gained by setting  $B_B^i \simeq 1$  ( $i = 1, \dots, 8$ ) for the  $\overline{B}^0 - B^0$  matrix elements of Eq.44 are partially offset by our ignorance of  $f_B$ .

## 4 Numerical result

In this section, we will give the numerical discussions and compare our results with experimental data. Before presenting the numerical results, we list the input parameters that are used in our discussions. For the CKM matrix elements, we use the Wolfenstein-parametrization with parameters  $A, \lambda, \rho, \eta$ . The SM-parameters are set as:  $G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}$ ,  $\alpha_s(m_w) = 0.12$ ,  $\alpha_s(m_b) = 0.22$ ,  $\alpha_s(m_c) = 0.34$ ,  $A = 0.80$ ,  $\lambda = 0.22$ ,  $m_b(m_w) = 4.5 \text{GeV}$ ,  $m_c(m_w) = 1.3 \text{GeV}$ ,  $m_t(m_w) = 167 \text{GeV}$ ,  $f_B = 0.2 \text{GeV}$ ,  $f_K = 0.167 \text{GeV}$ . For parameters  $\rho, \eta$ , we have  $\rho = 0.36$ ,  $\eta = 0$ . The factor  $B_K^i$  are chosen as  $B_K^1 = B_K^2 = B_K^3 = B_K^4 = B_K^5 = B_K^6 = B_K^7 = B_K^8 = 0.7$ . Using above parameters, the SM-predictions on  $\Delta m_B$  and  $\Delta m_K$  are

$$\Delta m_B(\text{SM}) = 2.18 \times 10^{-13} \text{GeV}, \Delta m_K(\text{SM}) = 2.89 \times 10^{-15} \text{GeV}.$$

At present, the experimental results are

$$\Delta m_B = (3.10 \pm 0.1) \times 10^{-13} \text{GeV}, \Delta m_K = (3.491 \pm 0.009) \times 10^{-15} \text{GeV}.$$

For the supersymmetric model with minimal flavor violation, the free parameters to be input are chosen as follows:  $\tan \beta = \frac{v_2}{v_1}$ ,  $m_{H^-}$ ,  $m_{\kappa_\lambda^-}$ ,  $m_{\tilde{U}_\alpha^i}$ ,  $m_{\tilde{B}_\alpha}$ ,  $m_{\tilde{D}_\alpha}$  ( $\alpha, \lambda=1, 2$ ) and the mixing matrix

$$\mathcal{Z}_{\tilde{U}^I} = \begin{pmatrix} \cos \xi_{\tilde{U}^I} & \sin \xi_{\tilde{U}^I} \\ -\sin \xi_{\tilde{U}^I} & \cos \xi_{\tilde{U}^I} \end{pmatrix}, \quad (45)$$

$$\mathcal{Z}_{\tilde{B}} = \begin{pmatrix} \cos \zeta_{\tilde{B}} & \sin \zeta_{\tilde{B}} \\ -\sin \zeta_{\tilde{B}} & \cos \zeta_{\tilde{B}} \end{pmatrix}, \quad (46)$$

$$\mathcal{Z}_{\tilde{D}} = \begin{pmatrix} \cos \zeta_{\tilde{D}} & \sin \zeta_{\tilde{D}} \\ -\sin \zeta_{\tilde{D}} & \cos \zeta_{\tilde{D}} \end{pmatrix}. \quad (47)$$

As for the mixing matrices of charginos  $\mathcal{Z}_\pm$ , they can be fixed by the values of  $\tan \beta$  and  $m_{\kappa_i^-}$ . In the numerical calculation, we assume that only one scalar quark is light and other heavy scalar quarks are taken as  $m_{\tilde{D}_1} = 4.5\text{TeV}$ ,  $m_{\tilde{B}_1} = 4.7\text{TeV}$ ,  $m_{\tilde{D}_2} = 4.6\text{TeV}$ ,  $m_{\tilde{B}_2} = 4.8\text{TeV}$ ,  $m_{\tilde{U}_1^1} = 4.1\text{TeV}$ ,  $m_{\tilde{U}_2^1} = 4.9\text{TeV}$ ,  $m_{\tilde{U}_1^2} = 4.05\text{TeV}$ ,  $m_{\tilde{U}_2^2} = 4.95\text{TeV}$  and  $m_{\tilde{U}_2^3} = 2.1\text{TeV}$ . For the heavy chargino, we set  $m_{\chi_2^-} = 2.2\text{TeV}$ . In order to suppress the number of free parameters, we assume the mixing angles to be equal  $\xi_{\tilde{U}^I} = \zeta_{\tilde{B}} = \zeta_{\tilde{D}}$  and focus on small value of  $\tan \xi_{\tilde{U}^I}$ .<sup>3</sup>

We obtain the dependence of  $\Delta m_B$  on the lighter scalar top quark mass with  $m_{\chi_1^-} = 110\text{GeV}$ ,  $m_{\tilde{g}} = 300\text{GeV}$ ,  $\tan \xi_{\tilde{U}^I} = \tan \zeta_{\tilde{D}} = \tan \zeta_{\tilde{B}} = 0$  and  $\tan \beta = 1, 5, 30$ . We find that as the lighter scalar top mass is greater than 300 GeV, the dependence is very mild, namely  $\Delta m_B$  almost does not change at all as  $m_{\tilde{U}_1^3}$  increases further and results with and without the gluino contributions only deviate by a constant of about  $0.3 \sim 1.0 \times 10^{-13}$  GeV depending on  $\tan \beta$  value.

The dependence of  $\Delta m_B$  on the lighter chargino mass is similar to that on the lighter stop mass. With  $m_{\tilde{U}_1^3} = 150\text{GeV}$ ,  $m_{\tilde{g}} = 300\text{GeV}$ ,  $\tan \xi_{\tilde{U}^I} = \tan \zeta_{\tilde{D}} = \tan \zeta_{\tilde{B}} = 0$  and  $\tan \beta = 1, 5, 30$ , as the chargino mass is greater than 200 GeV, the dependence is very mild, namely  $\Delta m_B$  almost does not change at all as  $m_{\chi_1^-}$  increases further and the results with and without the gluino contributions only deviate by a constant of  $0.2 \sim 0.8 \times 10^{-13}$  GeV depending on the  $\tan \beta$  value.

In Fig.9, we plot the dependence of  $\Delta m_B$  on gluino mass  $m_{\tilde{g}}$  with  $m_{\chi_1^-} = 110$  GeV,  $m_{\tilde{U}_1^3} = 150$  GeV,  $\tan \xi_{\tilde{U}^I} = \tan \zeta_{\tilde{D}} = \tan \zeta_{\tilde{B}} = 0$  and  $\tan \beta = 1, 5, 30$ . We find that  $\Delta m_B$  more sensitively depends on the gluino mass. Obviously, the results including NLO corrections from gluino are closer to the data than that without gluino contributions. It is also noted that as  $\tan \beta \sim 1$ , the data favors heavier gluino, i.e.  $m_{\tilde{g}}$  is greater than a few TeV's. But for  $\tan \beta \geq 5$ , the data favors  $m_{\tilde{g}} \sim 400 \sim 600$  GeV and the dependence of  $\Delta m_B$  is no longer sensitive to  $\tan \beta$ .

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<sup>3</sup>As free parameters, they can vary in the range  $-\frac{\pi}{4} \leq \xi_{\tilde{U}^I}, \zeta_{\tilde{B}}, \zeta_{\tilde{D}} \leq \frac{\pi}{4}$

For the case that the mixing between left- and right- squarks is non-zero, we take  $\tan \xi_{\tilde{U}I} = \tan \zeta_{\tilde{D}} = \tan \zeta_{\tilde{B}} = 0.1$  and plot  $\Delta m_B$  versus  $m_{\tilde{g}}$  in Fig.10. The situation is very similar to the discussions given above.

Now, we turn to the  $K^0 - \bar{K}^0$  mixing. In Fig.11, we plot the  $\Delta m_K$  versus the mass of gluino with other parameters being set as  $m_{\chi_1^-} = 413\text{GeV}$ ,  $m_{\tilde{U}_1^3} = 150\text{GeV}$ ,  $\tan \xi_{\tilde{U}I} = \tan \zeta_{\tilde{D}} = \tan \zeta_{\tilde{B}} = 0$  and  $\tan \beta = 1.5, 5, 30$  respectively. From these figures, we find that  $\Delta m_K$  is modified obviously when  $m_{\tilde{g}}$  varies.

It is noted that we assumed  $m_{\tilde{B}_1} \gg m_{\tilde{U}_1^3}$  in the above numerical computations, at present a possibility  $m_{\tilde{B}_1} \sim m_{\tilde{U}_1^3}$  is widely considered. We have re-calculated the resultant dependence of  $\Delta m_B$  and  $\Delta m_K$  on the gluino mass with  $m_{\tilde{B}_1} = m_{\tilde{U}_1^3} = 150\text{GeV}$  as input. Our numerical results show that for smaller gluino mass of about 300 GeV, the changes from that with larger  $m_{\tilde{B}_1}$  are very small and completely negligible. When the gluino mass turns larger, we find that the curves drop a bit faster. Concretely, as gluino mass reaches a region of about 3 TeV, the value of  $\Delta m_B$  is about 1.5% smaller than that with  $m_{\tilde{B}_1} = 4.7\text{ TeV}$ , and  $\Delta m_K$  is only suppressed by a factor of less than 1%.

From the above numerical analysis, we find that the gluino corrections cannot be neglected even when the gluino mass is very heavy. In the general case, the gluino mediated corrections depend on the choice of the parameter space and must be taken into account seriously.

## 5 Conclusions

We analyze the gluino mediated corrections to  $B^0 - \bar{B}^0$  mixing systematically up to the Next-to-Leading Order in the supersymmetric extension of standard model with minimal flavor violation. In the general case, the gluino contributions are evident and cannot be neglected in the NLO QCD corrections. Our technique can be used in other rare B processes such as  $b \rightarrow s\gamma$ ,  $b \rightarrow sg$ ,  $b \rightarrow sZ$  and  $b \rightarrow se^+e^-$  in the total supersymmetric calculations. After the systematic analysis on the B- and K- systems, we can expect to extract some constraints on the supersymmetric parameter space.

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## Appendix

## A The functions in the one-loop calculations

The functions in the one loop integrals are

$$\begin{aligned} f_a(x_1, x_2, x_3, x_4) &= \sum_{i=1}^4 \frac{x_i^2 \ln x_i}{\prod_{j \neq i} (x_j - x_i)}, \\ f_b(x_1, x_2, x_3, x_4) &= \sum_{i=1}^4 \frac{x_i \ln x_i}{\prod_{j \neq i} (x_j - x_i)}, \end{aligned} \quad (48)$$

when  $x_3 = x_4 = 1$ , they turn back to

$$\begin{aligned} f_a(x_1, x_2, 1, 1) &= \left( \frac{x_1^2 \ln x_1}{(x_2 - x_1)(1 - x_1)^2} + \frac{x_2^2 \ln x_2}{(x_1 - x_2)(1 - x_2)^2} \right. \\ &\quad \left. + \frac{1}{(1 - x_1)(1 - x_2)} \right), \\ f_b(x_1, x_2, 1, 1) &= \left( \frac{x_1 \ln x_1}{(x_2 - x_1)(1 - x_1)^2} + \frac{x_2 \ln x_2}{(x_1 - x_2)(1 - x_2)^2} \right. \\ &\quad \left. + \frac{1}{(1 - x_1)(1 - x_2)} \right). \end{aligned} \quad (49)$$

## B The integrand functions of two loop

In this appendix, we give some necessary integrals that are used in the context. The five topological diagrams are drawn in Fig.7, the integrand functions are defined as

$$\begin{aligned} I_{(i),0}(m_1^2, m_2^2, m_3^2, m_4^2, m_5^2, m_6^2, m_7^2) &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{2\pi)^D} \frac{1}{A_{(i)}}, \\ I_{(i),1}^a(m_1^2, m_2^2, m_3^2, m_4^2, m_5^2, m_6^2, m_7^2) &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{2\pi)^D} \frac{k^2}{A_{(i)}}, \\ I_{(i),1}^b(m_1^2, m_2^2, m_3^2, m_4^2, m_5^2, m_6^2, m_7^2) &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{2\pi)^D} \frac{q^2}{A_{(i)}}, \\ I_{(i),1}^c(m_1^2, m_2^2, m_3^2, m_4^2, m_5^2, m_6^2, m_7^2) &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{2\pi)^D} \frac{(k+q)^2}{A_{(i)}}, \\ I_{(i),2}^a(m_1^2, m_2^2, m_3^2, m_4^2, m_5^2, m_6^2, m_7^2) &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{2\pi)^D} \frac{k^4}{A_{(i)}}, \\ I_{(i),2}^b(m_1^2, m_2^2, m_3^2, m_4^2, m_5^2, m_6^2, m_7^2) &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{2\pi)^D} \frac{q^4}{A_{(i)}}, \end{aligned}$$

$$\begin{aligned}
I_{(i),2}^c(m_1^2, m_2^2, m_3^2, m_4^2, m_5^2, m_6^2, m_7^2) &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{2\pi)^D} \frac{(k+q)^4}{A_{(i)}}, \\
I_{(i),2}^d(m_1^2, m_2^2, m_3^2, m_4^2, m_5^2, m_6^2, m_7^2) &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{2\pi)^D} \frac{k^2 q^2}{A_{(i)}}, \\
I_{(i),2}^e(m_1^2, m_2^2, m_3^2, m_4^2, m_5^2, m_6^2, m_7^2) &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{2\pi)^D} \frac{k^2 (k+q)^2}{A_{(i)}}, \\
I_{(i),2}^f(m_1^2, m_2^2, m_3^2, m_4^2, m_5^2, m_6^2, m_7^2) &= \int \frac{d^D k}{(2\pi)^D} \frac{d^D q}{2\pi)^D} \frac{(k+q)^2 q^2}{A_{(i)}},
\end{aligned} \tag{50}$$

where the definitions of  $A_{(i)}$  are

$$\begin{aligned}
A_{(a)} &= (k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_3^2)((k+q)^2 - m_4^2)(q^2 - m_5^2)(q^2 - m_6^2)(q^2 - m_7^2), \\
A_{(b)} &= (k^2 - m_1^2)(k^2 - m_2^2)((k+q)^2 - m_3^2)((k+q)^2 - m_4^2)(q^2 - m_5^2)(q^2 - m_6^2)(q^2 - m_7^2), \\
A_{(c)} &= (k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_3^2)(k^2 - m_4^2)(k^2 - m_5^2)((k+q)^2 - m_6^2)(q^2 - m_7^2), \\
A_{(d)} &= (k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_3^2)(k^2 - m_4^2)((k+q)^2 - m_5^2)(q^2 - m_6^2)(q^2 - m_7^2), \\
A_{(e)} &= (k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_3^2)((k+q)^2 - m_4^2)((k+q)^2 - m_5^2)(q^2 - m_6^2)(q^2 - m_7^2).
\end{aligned} \tag{51}$$

Here,  $i = a, b, c, d, e$  are the indices of the diagrams in Fig.7

The loop integrals for diagram A are decomposed as

$$\begin{aligned}
I_{(a),2}^a &= I_{(a),2}^{a,1} + (m_2^2 + m_3^2)I_{(a),1}^a - m_2^2 m_3^2 I_{(a),0}, \\
I_{(a),2}^b &= I_{(a),2}^{b,1} + (m_6^2 + m_7^2)I_{(a),1}^b - m_6^2 m_7^2 I_{(a),0}, \\
I_{(a),2}^c &= I_{(a),2}^{c,1} + m_4^2 I_{(a),1}^c, \\
I_{(a),2}^d &= I_{(a),2}^{d,1} + m_3^2 I_{(a),1}^b + m_7^2 I_{(a),1}^a - m_3^2 m_7^2 I_{(a),0}, \\
I_{(a),2}^e &= I_{(a),2}^{e,1} + m_3^2 I_{(a),1}^c + m_4^2 I_{(a),1}^a - m_3^2 m_4^2 I_{(a),0}, \\
I_{(a),2}^f &= I_{(a),2}^{f,1} + m_4^2 I_{(a),1}^b + m_7^2 I_{(a),1}^c - m_4^2 m_7^2 I_{(a),0}, \\
I_{(a),1}^a &= I_{(a),1}^{a,1} + m_3^2 I_{(a),0}, \\
I_{(a),1}^b &= I_{(a),1}^{b,1} + m_7^2 I_{(a),0}, \\
I_{(a),1}^c &= I_{(a),1}^{c,1} + m_4^2 I_{(a),0}
\end{aligned} \tag{52}$$



with

$$\begin{aligned}
I_{(a),2}^{a,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=5}^7 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \ln x_{\rho\mu} \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) - \mathcal{S}L_{i_2}(x_{1\rho}, x_{4\rho}) \right. \\
&\quad \left. + \left( 3 - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} - \ln^2 x_{\rho\mu} \right), \\
I_{(a),2}^{b,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=1}^3 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \ln x_{\rho\mu} \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) - \mathcal{S}L_{i_2}(x_{4\rho}, x_{5\rho}) \right. \\
&\quad \left. + \left( 3 - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} - \ln^2 x_{\rho\mu} \right), \\
I_{(a),2}^{c,1} &= \frac{1}{(4\pi)^4} \sum_{\rho_1=1}^3 \sum_{\rho_2=5}^7 \frac{m_{\rho_1}^4 m_{\rho_2}^2 + m_{\rho_1}^2 m_{\rho_2}^4}{\prod_{\sigma_1 \neq \rho_1} (m_{\sigma_1}^2 - m_{\rho_1}^2) \prod_{\sigma_2 \neq \rho_2} (m_{\sigma_2}^2 - m_{\rho_2}^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln(x_{\rho_1\mu} x_{\rho_2\mu}) \right. \\
&\quad \left. + \left( 2 - \gamma_E + \ln 4\pi \right) \ln(x_{\rho_1\mu} x_{\rho_2\mu}) - \frac{1}{2} \ln^2(x_{\rho_1\mu} x_{\rho_2\mu}) \right), \\
I_{(a),2}^{d,1} &= \frac{1}{m_5^2 - m_6^2} \frac{1}{(4\pi)^2} \sum_{\rho=1}^2 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \mathcal{S}L_{i_2}(x_{4\rho}, x_{5\rho}) - \mathcal{S}L_{i_2}(x_{4\rho}, x_{6\rho}) \right), \\
I_{(a),2}^{e,1} &= \frac{1}{(4\pi)^4} \sum_{\rho_1=1}^2 \sum_{\rho_2=5}^7 \frac{m_{\rho_1}^2 m_{\rho_2}^2}{\prod_{\sigma_1 \neq \rho_1} (m_{\sigma_1}^2 - m_{\rho_1}^2) \prod_{\sigma_2 \neq \rho_2} (m_{\sigma_2}^2 - m_{\rho_2}^2)} \left( - \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln(x_{\rho_1\mu} x_{\rho_2\mu}) \right. \\
&\quad \left. - \left( 2 - \gamma_E + \ln 4\pi \right) \ln(x_{\rho_1\mu} x_{\rho_2\mu}) + \frac{1}{2} \ln^2(x_{\rho_1\mu} x_{\rho_2\mu}) \right), \\
I_{(a),2}^{f,1} &= \frac{1}{(4\pi)^4} \sum_{\rho_1=1}^3 \sum_{\rho_2=5}^6 \frac{m_{\rho_1}^2 m_{\rho_2}^2}{\prod_{\sigma_1 \neq \rho_1} (m_{\sigma_1}^2 - m_{\rho_1}^2) \prod_{\sigma_2 \neq \rho_2} (m_{\sigma_2}^2 - m_{\rho_2}^2)} \left( - \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln(x_{\rho_1\mu} x_{\rho_2\mu}) \right. \\
&\quad \left. - \left( 2 - \gamma_E + \ln 4\pi \right) \ln(x_{\rho_1\mu} x_{\rho_2\mu}) + \frac{1}{2} \ln^2(x_{\rho_1\mu} x_{\rho_2\mu}) \right), \\
I_{(a),1}^{a,1} &= -\frac{1}{m_1^2 - m_2^2} \frac{1}{(4\pi)^4} \sum_{\rho=5}^7 \frac{1}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( m_1^2 \mathcal{S}L_{i_2}(x_{41}, x_{\rho 1}) - m_2^2 \mathcal{S}L_{i_2}(x_{42}, x_{\rho 2}) \right), \\
I_{(a),1}^{b,1} &= -\frac{1}{m_5^2 - m_6^2} \frac{1}{(4\pi)^4} \sum_{\rho=1}^3 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \mathcal{S}L_{i_2}(x_{4\rho}, x_{5\rho}) - \mathcal{S}L_{i_2}(x_{4\rho}, x_{6\rho}) \right), \\
I_{(a),1}^{c,1} &= \frac{1}{2(4\pi)^4} \sum_{\rho_1=1}^3 \sum_{\rho_2=5}^7 \frac{m_{\rho_1}^2 m_{\rho_2}^2}{\prod_{\sigma_1 \neq \rho_1} (m_{\sigma_1}^2 - m_{\rho_1}^2) \prod_{\sigma_2 \neq \rho_2} (m_{\sigma_2}^2 - m_{\rho_2}^2)} \ln^2(x_{\rho_1\mu} x_{\rho_2\mu}), \\
I_{(a),0} &= -\frac{1}{(4\pi)^4} \sum_{\rho_1=1}^3 \sum_{\rho_2=5}^7 \frac{m_{\rho_1}^2}{\prod_{\sigma_1 \neq \rho_1} (m_{\sigma_1}^2 - m_{\rho_1}^2) \prod_{\sigma_2 \neq \rho_2} (m_{\sigma_2}^2 - m_{\rho_2}^2)} \mathcal{S}L_{i_2}(x_{4\rho_1}, x_{\rho_2\rho_1}). \tag{53}
\end{aligned}$$

For the diagrams of class B, the loop integrals are decomposed as

$$\begin{aligned}
I_{(b),2}^a &= I_{(b),2}^{a,1} + (m_1^2 + m_2^2)I_{(b),1}^a - m_1^2 m_2^2 I_{(b),0}^a, \\
I_{(b),2}^b &= I_{(b),2}^{b,1} + (m_6^2 + m_7^2)I_{(b),1}^b - m_6^2 m_7^2 I_{(b),0}^b, \\
I_{(b),2}^c &= I_{(b),2}^{c,1} + (m_3^2 + m_4^2)I_{(b),1}^c - m_3^2 m_4^2 I_{(b),0}^c, \\
I_{(b),2}^d &= I_{(b),2}^{d,1} + m_2^2 I_{(b),1}^b + m_7^2 I_{(b),1}^a - m_2^2 m_7^2 I_{(b),0}^b, \\
I_{(b),2}^e &= I_{(b),2}^{e,1} + m_2^2 I_{(b),1}^c + m_4^2 I_{(b),1}^a - m_2^2 m_4^2 I_{(b),0}^c, \\
I_{(b),2}^f &= I_{(b),2}^{f,1} + m_4^2 I_{(b),1}^b + m_7^2 I_{(b),1}^c - m_4^2 m_7^2 I_{(b),0}^b, \\
I_{(b),1}^a &= I_{(b),1}^{a,1} + m_2^2 I_{(b),0}^a, \\
I_{(b),1}^b &= I_{(b),1}^{b,1} + m_7^2 I_{(b),0}^b, \\
I_{(b),1}^c &= I_{(b),1}^{c,1} + m_4^2 I_{(b),0}^c
\end{aligned} \tag{54}$$

and

$$\begin{aligned}
I_{(b),2}^{a,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=5}^7 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 2 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} \right. \\
&\quad \left. - \frac{m_3^2}{2(m_3^2 - m_4^2)} \ln^2(x_{3\mu} x_{\rho\mu}) + \frac{m_4^2}{2(m_3^2 - m_4^2)} \ln^2(x_{4\mu} x_{\rho\mu}) \right), \\
I_{(b),2}^{b,1} &= -\frac{1}{(4\pi)^4} \frac{1}{(m_1^2 - m_2^2)(m_3^2 - m_4^2)} \left( m_1^2 \left( \mathcal{S}L_{i_2}(x_{31}, x_{51}) - \mathcal{S}L_{i_2}(x_{41}, x_{51}) \right) \right. \\
&\quad \left. - m_2^2 \left( \mathcal{S}L_{i_2}(x_{32}, x_{52}) - \mathcal{S}L_{i_2}(x_{42}, x_{52}) \right) \right), \\
I_{(b),2}^{c,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=5}^7 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 2 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} \right. \\
&\quad \left. - \frac{m_1^2}{2(m_1^2 - m_2^2)} \ln^2(x_{1\mu} x_{\rho\mu}) + \frac{m_2^2}{2(m_1^2 - m_2^2)} \ln^2(x_{2\mu} x_{\rho\mu}) \right), \\
I_{(b),2}^{d,1} &= -\frac{1}{(4\pi)^4} \frac{1}{(m_3^2 - m_4^2)(m_5^2 - m_6^2)} \left( m_5^2 \left( \mathcal{S}L_{i_2}(x_{15}, x_{35}) - \mathcal{S}L_{i_2}(x_{15}, x_{45}) \right) \right. \\
&\quad \left. - m_6^2 \left( \mathcal{S}L_{i_2}(x_{16}, x_{36}) - \mathcal{S}L_{i_2}(x_{16}, x_{46}) \right) \right), \\
I_{(b),2}^{e,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=5}^7 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 3 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} \right.
\end{aligned}$$

$$\begin{aligned}
& -\ln^2 x_{\rho\mu} - \mathcal{S}L_{i_2}(x_{1\rho}, x_{3\rho}) \Big) , \\
I_{(b),2}^{f,1} &= -\frac{1}{(4\pi)^4} \frac{1}{(m_1^2 - m_2^2)(m_5^2 - m_6^2)} \Big( m_1^2 (\mathcal{S}L_{i_2}(x_{31}, x_{51}) - \mathcal{S}L_{i_2}(x_{31}, x_{61})) \\
& - m_2^2 (\mathcal{S}L_{i_2}(x_{32}, x_{52}) - \mathcal{S}L_{i_2}(x_{32}, x_{62})) \Big) , \\
I_{(b),1}^{a,1} &= -\frac{1}{(4\pi)^4} \frac{1}{m_3^2 - m_4^2} \sum_{\rho=5}^7 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \Big( \mathcal{S}L_{i_2}(x_{1\rho}, x_{3\rho}) - \mathcal{S}L_{i_2}(x_{1\rho}, x_{4\rho}) \Big) , \\
I_{(b),1}^{b,1} &= -\frac{1}{(4\pi)^4} \frac{1}{(m_1^2 - m_2^2)(m_3^2 - m_4^2)(m_5^2 - m_6^2)} \Big( m_1^2 (\mathcal{S}L_{i_2}(x_{31}, x_{51}) - \mathcal{S}L_{i_2}(x_{31}, x_{61}) \\
& - \mathcal{S}L_{i_2}(x_{41}, x_{51}) + \mathcal{S}L_{i_2}(x_{41}, x_{61})) - m_2^2 (\mathcal{S}L_{i_2}(x_{32}, x_{52}) - \mathcal{S}L_{i_2}(x_{32}, x_{62}) \\
& - \mathcal{S}L_{i_2}(x_{42}, x_{52}) + \mathcal{S}L_{i_2}(x_{42}, x_{62})) \Big) , \\
I_{(b),1}^{c,1} &= \frac{1}{(4\pi)^4} \frac{1}{m_1^2 - m_2^2} \sum_{\rho=5}^7 \frac{1}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \Big( m_1^2 \mathcal{S}L_{i_2}(x_{31}, x_{\rho 1}) - m_2^2 \mathcal{S}L_{i_2}(x_{32}, x_{\rho 2}) \Big) , \\
I_{(b),0} &= -\frac{1}{(4\pi)^4} \frac{1}{(m_1^2 - m_2^2)(m_3^2 - m_4^2)} \sum_{\rho=5}^7 \frac{1}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \Big( m_1^2 (\mathcal{S}L_{i_2}(x_{31}, x_{\rho 1}) - \mathcal{S}L_{i_2}(x_{41}, x_{\rho 1})) \\
& - m_2^2 (\mathcal{S}L_{i_2}(x_{32}, x_{\rho 2}) - \mathcal{S}L_{i_2}(x_{42}, x_{\rho 2})) \Big) . \tag{55}
\end{aligned}$$

The reduced formulae for the self energy insertion diagrams (class C) are:

$$\begin{aligned}
I_{(c),2}^a &= I_{(c),2}^{a,1} + (m_4^2 + m_5^2) I_{(c),1}^a - m_4^2 m_5^2 I_{(c),0} , \\
I_{(c),2}^b &= I_{(c),2}^{b,1} + m_7^2 I_{(c),1}^b , \\
I_{(c),2}^c &= I_{(c),2}^{c,1} + m_6^2 I_{(c),1}^c , \\
I_{(c),2}^d &= I_{(c),2}^{d,1} + m_5^2 I_{(c),1}^b + m_7^2 I_{(c),1}^a - m_5^2 m_7^2 I_{(c),0} , \\
I_{(c),2}^e &= I_{(c),2}^{e,1} + m_5^2 I_{(c),1}^c + m_6^2 I_{(c),1}^a - m_5^2 m_6^2 I_{(c),0} , \\
I_{(c),2}^f &= I_{(c),2}^{f,1} + m_6^2 I_{(c),1}^c + m_7^2 I_{(c),1}^b - m_6^2 m_7^2 I_{(c),0} , \\
I_{(c),1}^a &= I_{(c),1}^{a,1} + m_5^2 I_{(c),0} , \\
I_{(c),1}^b &= I_{(c),1}^{b,1} + m_7^2 I_{(c),0} , \\
I_{(c),1}^c &= I_{(c),1}^{c,1} + m_6^2 I_{(c),0} \tag{56}
\end{aligned}$$

and

$$\begin{aligned}
I_{(c),2}^{a,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=1}^3 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 3 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} \right. \\
&\quad \left. - \ln^2 x_{\rho\mu} - \mathcal{S}L_{i_2}(x_{6\rho}, x_{7\rho}) \right), \\
I_{(c),2}^{b,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=1}^5 \frac{m_\rho^4 m_6^2 + m_\rho^2 m_6^4}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln(x_{\rho\mu} x_{6\mu}) + \left( 2 - \gamma_E + \ln 4\pi \right) \ln(x_{\rho\mu} x_{6\mu}) \right. \\
&\quad \left. - \frac{1}{2} \ln^2(x_{\rho\mu} x_{6\mu}) \right), \\
I_{(c),2}^{c,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=1}^5 \frac{m_\rho^4 m_7^2 + m_\rho^2 m_7^4}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln(x_{\rho\mu} x_{7\mu}) + \left( 2 - \gamma_E + \ln 4\pi \right) \ln(x_{\rho\mu} x_{7\mu}) \right. \\
&\quad \left. - \frac{1}{2} \ln^2(x_{\rho\mu} x_{7\mu}) \right), \\
I_{(c),2}^{d,1} &= -\frac{1}{(4\pi)^4} \sum_{\rho=1}^4 \frac{m_\rho^2 m_6^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln(x_{\rho\mu} x_{6\mu}) + \left( 2 - \gamma_E + \ln 4\pi \right) \ln(x_{\rho\mu} x_{6\mu}) \right. \\
&\quad \left. - \frac{1}{2} \ln^2(x_{\rho\mu} x_{6\mu}) \right), \\
I_{(c),2}^{e,1} &= -\frac{1}{(4\pi)^4} \sum_{\rho=1}^4 \frac{m_\rho^2 m_7^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln(x_{\rho\mu} x_{7\mu}) + \left( 2 - \gamma_E + \ln 4\pi \right) \ln(x_{\rho\mu} x_{7\mu}) \right. \\
&\quad \left. - \frac{1}{2} \ln^2(x_{\rho\mu} x_{7\mu}) \right), \\
I_{(c),2}^{f,1} &= 0, \\
I_{(c),1}^{a,1} &= -\frac{1}{(4\pi)^4} \sum_{\rho=1}^4 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 3 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} \right. \\
&\quad \left. - \ln^2 x_{\rho\mu} - \mathcal{S}L_{i_2}(x_{6\rho}, x_{7\rho}) \right), \\
I_{(c),1}^{b,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=1}^5 \frac{m_\rho^2 m_6^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln(x_{\rho\mu} x_{6\mu}) + \left( 2 - \gamma_E + \ln 4\pi \right) \ln(x_{\rho\mu} x_{6\mu}) \right. \\
&\quad \left. - \frac{1}{2} \ln^2(x_{\rho\mu} x_{6\mu}) \right), \\
I_{(c),1}^{c,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=1}^5 \frac{m_\rho^2 m_7^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln(x_{\rho\mu} x_{7\mu}) + \left( 2 - \gamma_E + \ln 4\pi \right) \ln(x_{\rho\mu} x_{7\mu}) \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2} \ln^2(x_{\rho\mu} x_{7\mu}) \Big) , \\
I_{(c),0} = & \frac{1}{(4\pi)^4} \sum_{\rho=1}^5 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 3 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} \right. \\
& \left. - \ln^2 x_{\rho\mu} - \mathcal{S}L_{i_2}(x_{6\rho}, x_{7\rho}) \right) .
\end{aligned} \tag{57}$$

In a similar way, the formulae for vertex insertion diagrams (class D) are decomposed into

$$\begin{aligned}
I_{(d),2}^a &= I_{(d),2}^{a,1} + (m_3^2 + m_4^2) I_{(d),1}^a - m_3^2 m_4^2 I_{(d),0} , \\
I_{(d),2}^b &= I_{(d),2}^{b,1} + (m_6^2 + m_7^2) I_{(d),1}^b - m_6^2 m_7^2 I_{(d),0} , \\
I_{(d),2}^c &= I_{(d),2}^{c,1} + m_5^2 I_{(d),1}^c , \\
I_{(d),2}^d &= I_{(d),2}^{d,1} + m_4^2 I_{(d),1}^b + m_7^2 I_{(d),1}^a - m_4^2 m_7^2 I_{(d),0} , \\
I_{(d),2}^e &= I_{(d),2}^{e,1} + m_4^2 I_{(d),1}^c + m_5^2 I_{(d),1}^a - m_4^2 m_5^2 I_{(d),0} , \\
I_{(d),2}^f &= I_{(d),2}^{f,1} + m_5^2 I_{(d),1}^b + m_7^2 I_{(d),1}^c - m_5^2 m_7^2 I_{(d),0} , \\
I_{(d),1}^a &= I_{(d),1}^{a,1} + m_4^2 I_{(d),0} , \\
I_{(d),1}^b &= I_{(d),1}^{b,1} + m_7^2 I_{(d),0} , \\
I_{(d),1}^c &= I_{(d),1}^{c,1} + m_5^2 I_{(d),0}
\end{aligned} \tag{58}$$

with

$$\begin{aligned}
I_{(d),2}^{a,1} &= \frac{1}{(4\pi)^4} \frac{1}{(m_1^2 - m_2^2)(m_6^2 - m_7^2)} \left( m_1^2 \left( \mathcal{S}L_{i_2}(x_{51}, x_{61}) - \mathcal{S}L_{i_2}(x_{51}, x_{71}) \right) \right. \\
& \left. - m_2^2 \left( \mathcal{S}L_{i_2}(x_{52}, x_{62}) - \mathcal{S}L_{i_2}(x_{52}, x_{72}) \right) \right) , \\
I_{(d),2}^{b,1} &= -\frac{1}{(4\pi)^4} \sum_{\rho=1}^4 \frac{m_\rho^2 m_5^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln(x_{\rho\mu} x_{5\mu}) + \left( 2 - \gamma_E + \ln 4\pi \right) \ln(x_{\rho\mu} x_{5\mu}) \right. \\
& \left. + \frac{1}{2} \ln^2(x_{\rho\mu} x_{5\mu}) \right) , \\
I_{(d),2}^{c,1} &= -\frac{1}{(4\pi)^4} \frac{1}{m_6^2 - m_7^2} \sum_{\rho=1}^4 \frac{1}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( m_\rho^2 m_6^2 (m_\rho^2 + m_6^2) - m_\rho^2 m_7^2 (m_\rho^2 + m_7^2) \right) \left( \frac{1}{\varepsilon} - \gamma_E \right. \right. \\
& \left. \left. + \ln(4\pi) \right) \ln x_{\rho\mu} + m_\rho^2 m_6^2 (m_\rho^2 + m_6^2) \left( (2 - \gamma_E + \ln 4\pi) \ln(x_{\rho\mu} x_{6\mu}) - \frac{1}{2} \ln^2(x_{\rho\mu} x_{6\mu}) \right) \right.
\end{aligned}$$

$$\begin{aligned}
& -m_\rho^2 m_7^2 (m_\rho^2 + m_7^2) \left( (2 - \gamma_E + \ln 4\pi) \ln(x_{\rho\mu} x_{7\mu}) - \frac{1}{2} \ln^2(x_{\rho\mu} x_{7\mu}) \right), \\
I_{(d),2}^{d,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=1}^3 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 3 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} \right. \\
& \quad \left. - \ln^2 x_{\rho\mu} - \mathcal{S}L_{i_2}(x_{5\rho}, x_{6\rho}) \right), \\
I_{(d),2}^{e,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=1}^3 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 2 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} \right. \\
& \quad \left. - \frac{m_6^2 \ln^2(x_{6\mu} x_{\rho\mu}) - m_7^2 \ln^2(x_{7\mu} x_{\rho\mu})}{2(m_6^2 - m_7^2)} \right), \\
I_{(d),2}^{f,1} &= -\frac{1}{(4\pi)^4} \sum_{\rho=1}^3 \frac{m_6^2 m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln(x_{6\mu} x_{\rho\mu}) + \left( 2 - \gamma_E + \ln 4\pi \right) \ln(x_{6\mu} x_{\rho\mu}) \right. \\
& \quad \left. + \frac{1}{2} \ln^2(x_{6\mu} x_{\rho\mu}) \right), \\
I_{(d),1}^{a,1} &= -\frac{1}{(4\pi)^4} \frac{1}{m_6^2 - m_7^2} \sum_{\rho=1}^3 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \mathcal{S}L_{i_2}(x_{5\rho}, x_{6\rho}) - \mathcal{S}L_{i_2}(x_{5\rho}, x_{7\rho}) \right), \\
I_{(d),1}^{b,1} &= -\frac{1}{(4\pi)^4} \sum_{\rho=1}^4 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 3 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} - \ln^2 x_{\rho\mu} \right), \\
I_{(d),1}^{c,1} &= -\frac{1}{(4\pi)^4} \sum_{\rho=1}^4 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 2 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} \right. \\
& \quad \left. - \frac{m_6^2 \ln^2(x_{6\mu} x_{\rho\mu}) - m_7^2 \ln^2(x_{7\mu} x_{\rho\mu})}{2(m_6^2 - m_7^2)} \right), \\
I_{(d),0} &= \frac{1}{(4\pi)^4} \frac{1}{m_6^2 - m_7^2} \sum_{\rho=1}^4 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \mathcal{S}L_{i_2}(x_{5\rho}, x_{6\rho}) - \mathcal{S}L_{i_2}(x_{5\rho}, x_{7\rho}) \right). \tag{59}
\end{aligned}$$

For the topological class E, the formulae are written as

$$\begin{aligned}
I_{(e),2}^a &= I_{(e),2}^{a,1} + (m_2^2 + m_3^2) I_{(e),1}^a - m_2^2 m_3^2 I_{(e),0}, \\
I_{(e),2}^b &= I_{(e),2}^{b,1} + (m_6^2 + m_7^2) I_{(e),1}^b - m_6^2 m_7^2 I_{(e),0}, \\
I_{(e),2}^c &= I_{(e),2}^{c,1} + (m_4^2 + m_5^2) I_{(e),1}^c - m_4^2 m_5^2 I_{(e),0}, \\
I_{(e),2}^d &= I_{(e),2}^{d,1} + m_3^2 I_{(e),1}^b + m_7^2 I_{(e),1}^a - m_3^2 m_7^2 I_{(e),0},
\end{aligned}$$

$$\begin{aligned}
I_{(e),2}^e &= I_{(e),2}^{e,1} + m_3^2 I_{(e),1}^c + m_5^2 I_{(e),1}^a - m_3^2 m_5^2 I_{(e),0} , \\
I_{(e),2}^f &= I_{(e),2}^{f,1} + m_5^2 I_{(e),1}^c + m_7^2 I_{(e),1}^b - m_5^2 m_7^2 I_{(e),0} , \\
I_{(e),1}^a &= I_{(e),1}^{a,1} + m_3^2 I_{(e),0} , \\
I_{(e),1}^b &= I_{(e),1}^{b,1} + m_7^2 I_{(e),0} , \\
I_{(e),1}^c &= I_{(e),1}^{c,1} + m_5^2 I_{(e),0}
\end{aligned} \tag{60}$$

and

$$\begin{aligned}
I_{(e),2}^{a,1} &= -\frac{1}{(4\pi)^4} \frac{1}{(m_4^2 - m_5^2)(m_6^2 - m_7^2)} \left( m_6^2 \left( \mathcal{S}L_{i_2}(x_{16}, x_{46}) - \mathcal{S}L_{i_2}(x_{16}, x_{56}) \right) \right. \\
&\quad \left. - m_7^2 \left( \mathcal{S}L_{i_2}(x_{17}, x_{47}) - \mathcal{S}L_{i_2}(x_{17}, x_{57}) \right) \right) , \\
I_{(e),2}^{b,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=1}^3 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 2 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} \right. \\
&\quad \left. - \frac{m_4^2 \ln^2(x_{4\mu} x_{\rho\mu}) - m_5^2 \ln^2(x_{5\mu} x_{\rho\mu})}{2(m_4^2 - m_5^2)} \right) , \\
I_{(e),2}^{c,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=1}^3 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 2 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} \right. \\
&\quad \left. - \frac{m_6^2 \ln^2(x_{6\mu} x_{\rho\mu}) - m_7^2 \ln^2(x_{7\mu} x_{\rho\mu})}{2(m_6^2 - m_7^2)} \right) , \\
I_{(e),2}^{d,1} &= -\frac{1}{(4\pi)^4} \frac{1}{(m_1^2 - m_2^2)(m_4^2 - m_5^2)} \left( m_1^2 \left( \mathcal{S}L_{i_2}(x_{41}, x_{61}) - \mathcal{S}L_{i_2}(x_{51}, x_{61}) \right) \right. \\
&\quad \left. - m_2^2 \left( \mathcal{S}L_{i_2}(x_{42}, x_{62}) - \mathcal{S}L_{i_2}(x_{52}, x_{62}) \right) \right) , \\
I_{(e),2}^{e,1} &= -\frac{1}{(4\pi)^4} \frac{1}{(m_1^2 - m_2^2)(m_6^2 - m_7^2)} \left( m_1^2 \left( \mathcal{S}L_{i_2}(x_{41}, x_{61}) - \mathcal{S}L_{i_2}(x_{41}, x_{71}) \right) \right. \\
&\quad \left. - m_2^2 \left( \mathcal{S}L_{i_2}(x_{42}, x_{62}) - \mathcal{S}L_{i_2}(x_{42}, x_{72}) \right) \right) , \\
I_{(e),2}^{f,1} &= \frac{1}{(4\pi)^4} \sum_{\rho=1}^3 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \left( \left( \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) \right) \ln x_{\rho\mu} + \left( 3 - \gamma_E + \ln 4\pi \right) \ln x_{\rho\mu} \right. \\
&\quad \left. - \ln^2 x_{\rho\mu} - \mathcal{S}L_{i_2}(x_{4\rho}, x_{6\rho}) \right) , \\
I_{(e),1}^{a,1} &= -\frac{1}{(4\pi)^4} \frac{1}{(m_1^2 - m_2^2)(m_4^2 - m_5^2)(m_6^2 - m_7^2)} \left( m_1^2 \left( \mathcal{S}L_{i_2}(x_{41}, x_{61}) - \mathcal{S}L_{i_2}(x_{41}, x_{71}) \right) \right.
\end{aligned}$$

$$\begin{aligned}
& -\mathcal{S}L_{i_2}(x_{51}, x_{61}) + \mathcal{S}L_{i_2}(x_{51}, x_{71}) \Big) - m_2^2 \Big( \mathcal{S}L_{i_2}(x_{42}, x_{62}) - \mathcal{S}L_{i_2}(x_{42}, x_{72}) \\
& -\mathcal{S}L_{i_2}(x_{52}, x_{62}) + \mathcal{S}L_{i_2}(x_{52}, x_{72}) \Big) \Big) , \\
I_{(e),1}^{b,1} &= -\frac{1}{(4\pi)^4} \frac{1}{(m_4^2 - m_5^2)} \sum_{\rho=1}^3 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \Big( \mathcal{S}L_{i_2}(x_{4\rho}, x_{6\rho}) - \mathcal{S}L_{i_2}(x_{5\rho}, x_{6\rho}) \Big) , \\
I_{(e),1}^{c,1} &= -\frac{1}{(4\pi)^4} \frac{1}{(m_6^2 - m_7^2)} \sum_{\rho=1}^3 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \Big( \mathcal{S}L_{i_2}(x_{4\rho}, x_{6\rho}) - \mathcal{S}L_{i_2}(x_{4\rho}, x_{7\rho}) \Big) , \\
I_{(e),0} &= -\frac{1}{(4\pi)^4} \frac{1}{(m_4^2 - m_5^2)(m_6^2 - m_7^2)} \sum_{\rho=1}^3 \frac{m_\rho^2}{\prod_{\sigma \neq \rho} (m_\sigma^2 - m_\rho^2)} \Big( \mathcal{S}L_{i_2}(x_{4\rho}, x_{6\rho}) - \mathcal{S}L_{i_2}(x_{4\rho}, x_{7\rho}) \\
& -\mathcal{S}L_{i_2}(x_{5\rho}, x_{6\rho}) + \mathcal{S}L_{i_2}(x_{5\rho}, x_{7\rho}) \Big) . \tag{61}
\end{aligned}$$

When we compute the amplitude of  $\Delta B = 2$  processes, the class C and class D contain the ultraviolet divergence and we adopt the  $\overline{MS}$ - scheme to remove them. After the above step, the functions that appear in the effective Hamiltonian are

$$\begin{aligned}
F_{(a),2}^{a,1} &= \sum_{\rho=5}^7 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \Big( -\mathcal{S}L_{i_2}(x_{1\rho}, x_{4\rho}) + \ln x_{\rho\mu} - \ln^2 x_{\rho\mu} \Big) , \\
F_{(a),2}^{b,1} &= \sum_{\rho=1}^3 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \Big( -\mathcal{S}L_{i_2}(x_{4\rho}, x_{5\rho}) + \ln x_{\rho\mu} - \ln^2 x_{\rho\mu} \Big) , \\
F_{(a),2}^{c,1} &= -\frac{1}{2} \sum_{\rho_1=1}^3 \sum_{\rho_2=5}^7 \frac{x_{\rho_1\mu}^2 x_{\rho_2\mu} + x_{\rho_1\mu} x_{\rho_2\mu}^2}{\prod_{\sigma_1 \neq \rho_1} (x_{\sigma_1\mu} - x_{\rho_1\mu}) \prod_{\sigma_2 \neq \rho_2} (x_{\sigma_2\mu} - x_{\rho_2\mu})} \ln^2(x_{\rho_1\mu} x_{\rho_2\mu}) , \\
F_{(a),2}^{d,1} &= \frac{1}{x_{5\mu} - x_{6\mu}} \sum_{\rho=1}^2 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \Big( \mathcal{S}L_{i_2}(x_{4\rho}, x_{5\rho}) - \mathcal{S}L_{i_2}(x_{4\rho}, x_{6\rho}) \Big) , \\
F_{(a),2}^{e,1} &= \frac{1}{2} \sum_{\rho_1=1}^2 \sum_{\rho_2=5}^7 \frac{x_{\rho_1\mu} x_{\rho_2\mu}}{\prod_{\sigma_1 \neq \rho_1} (x_{\sigma_1\mu} - x_{\rho_1\mu}) \prod_{\sigma_2 \neq \rho_2} (x_{\sigma_2\mu} - x_{\rho_2\mu})} \ln^2(x_{\rho_1\mu} x_{\rho_2\mu}) , \\
F_{(a),2}^{f,1} &= \frac{1}{2} \sum_{\rho_1=1}^3 \sum_{\rho_2=5}^6 \frac{x_{\rho_1\mu} x_{\rho_2\mu}}{\prod_{\sigma_1 \neq \rho_1} (x_{\sigma_1\mu} - x_{\rho_1\mu}) \prod_{\sigma_2 \neq \rho_2} (x_{\sigma_2\mu} - x_{\rho_2\mu})} \ln^2(x_{\rho_1\mu} x_{\rho_2\mu}) , \\
F_{(a),1}^{a,1} &= -\frac{1}{x_{1\mu} - x_{2\mu}} \sum_{\rho=5}^7 \frac{1}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \Big( x_{1\mu} \mathcal{S}L_{i_2}(x_{41}, x_{\rho 1}) - x_{2\mu} \mathcal{S}L_{i_2}(x_{42}, x_{\rho 2}) \Big) ,
\end{aligned}$$



$$\begin{aligned}
F_{(a),1}^{b,1} &= -\frac{1}{x_{5\mu} - x_{6\mu}} \sum_{\rho=1}^3 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \mathcal{S}L_{i_2}(x_{4\rho}, x_{5\rho}) - \mathcal{S}L_{i_2}(x_{4\rho}, x_{6\rho}) \right), \\
F_{(a),1}^{c,1} &= \frac{1}{2} \sum_{\rho_1=1}^3 \sum_{\rho_2=5}^7 \frac{x_{\rho_1\mu} x_{\rho_2\mu}}{\prod_{\sigma_1 \neq \rho_1} (x_{\sigma_1\mu} - x_{\rho_1\mu}) \prod_{\sigma_2 \neq \rho_2} (x_{\sigma_2\mu} - x_{\rho_2\mu})} \ln^2(x_{\rho_1\mu} x_{\rho_2\mu}), \\
F_{(a),0} &= -\sum_{\rho_1=1}^3 \sum_{\rho_2=5}^7 \frac{x_{\rho_1\mu}}{\prod_{\sigma_1 \neq \rho_1} (x_{\sigma_1\mu} - x_{\rho_1\mu}) \prod_{\sigma_2 \neq \rho_2} (x_{\sigma_2\mu} - x_{\rho_2\mu})} \mathcal{S}L_{i_2}(x_{4\rho_1}, x_{\rho_2\rho_1}), \tag{62}
\end{aligned}$$

and

$$\begin{aligned}
F_{(b),2}^{a,1} &= \sum_{\rho=5}^7 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( -\frac{x_{3\mu}}{2(x_{3\mu} - x_{4\mu})} \ln^2(x_{3\mu} x_{\rho\mu}) + \frac{x_{4\mu}}{2(x_{3\mu} - x_{4\mu})} \ln^2(x_{4\mu} x_{\rho\mu}) \right), \\
F_{(b),2}^{b,1} &= -\frac{1}{(x_{1\mu} - x_{2\mu})(x_{3\mu} - x_{4\mu})} \left( x_{1\mu} \left( \mathcal{S}L_{i_2}(x_{31}, x_{51}) - \mathcal{S}L_{i_2}(x_{41}, x_{51}) \right) \right. \\
&\quad \left. - x_{2\mu} \left( \mathcal{S}L_{i_2}(x_{32}, x_{52}) - \mathcal{S}L_{i_2}(x_{42}, x_{52}) \right) \right), \\
F_{(b),2}^{c,1} &= \sum_{\rho=5}^7 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( -\frac{x_{1\mu}}{2(x_{1\mu} - x_{2\mu})} \ln^2(x_{1\mu} x_{\rho\mu}) + \frac{x_{2\mu}}{2(x_{1\mu} - x_{2\mu})} \ln^2(x_{2\mu} x_{\rho\mu}) \right), \\
F_{(b),2}^{d,1} &= -\frac{1}{(x_{3\mu} - x_{4\mu})(x_{5\mu} - x_{6\mu})} \left( x_{5\mu} \left( \mathcal{S}L_{i_2}(x_{15}, x_{35}) - \mathcal{S}L_{i_2}(x_{15}, x_{45}) \right) \right. \\
&\quad \left. - x_{6\mu} \left( \mathcal{S}L_{i_2}(x_{16}, x_{36}) - \mathcal{S}L_{i_2}(x_{16}, x_{46}) \right) \right), \\
F_{(b),2}^{e,1} &= \sum_{\rho=5}^7 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \ln x_{\rho\mu} - \ln^2 x_{\rho\mu} - \mathcal{S}L_{i_2}(x_{1\rho}, x_{3\rho}) \right), \\
F_{(b),2}^{f,1} &= -\frac{1}{(x_{1\mu} - x_{2\mu})(x_{5\mu} - x_{6\mu})} \left( x_{1\mu} \left( \mathcal{S}L_{i_2}(x_{31}, x_{51}) - \mathcal{S}L_{i_2}(x_{31}, x_{61}) \right) \right. \\
&\quad \left. - x_{2\mu} \left( \mathcal{S}L_{i_2}(x_{32}, x_{52}) - \mathcal{S}L_{i_2}(x_{32}, x_{62}) \right) \right), \\
F_{(b),1}^{a,1} &= -\frac{1}{x_{3\mu} - x_{4\mu}} \sum_{\rho=5}^7 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \mathcal{S}L_{i_2}(x_{1\rho}, x_{3\rho}) - \mathcal{S}L_{i_2}(x_{1\rho}, x_{4\rho}) \right), \\
F_{(b),1}^{b,1} &= -\frac{1}{(x_{1\mu} - x_{2\mu})(x_{3\mu} - x_{4\mu})(x_{5\mu} - x_{6\mu})} \left( x_{1\mu} \left( \mathcal{S}L_{i_2}(x_{31}, x_{51}) - \mathcal{S}L_{i_2}(x_{31}, x_{61}) \right) \right. \\
&\quad - \mathcal{S}L_{i_2}(x_{41}, x_{51}) + \mathcal{S}L_{i_2}(x_{41}, x_{61}) - x_{2\mu} \left( \mathcal{S}L_{i_2}(x_{32}, x_{52}) - \mathcal{S}L_{i_2}(x_{32}, x_{62}) \right) \\
&\quad \left. - \mathcal{S}L_{i_2}(x_{42}, x_{52}) + \mathcal{S}L_{i_2}(x_{42}, x_{62}) \right),
\end{aligned}$$

$$\begin{aligned}
F_{(b),1}^{c,1} &= \frac{1}{x_{1\mu} - x_{2\mu}} \sum_{\rho=5}^7 \frac{1}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( x_{1\mu} \mathcal{S}L_{i_2}(x_{31}, x_{\rho 1}) - x_{2\mu} \mathcal{S}L_{i_2}(x_{32}, x_{\rho 2}) \right), \\
F_{(b),0} &= -\frac{1}{(x_{1\mu} - x_{2\mu})(x_{3\mu} - x_{4\mu})} \sum_{\rho=5}^7 \frac{1}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( x_{1\mu} \left( \mathcal{S}L_{i_2}(x_{31}, x_{\rho 1}) - \mathcal{S}L_{i_2}(x_{41}, x_{\rho 1}) \right) \right. \\
&\quad \left. - x_{2\mu} \left( \mathcal{S}L_{i_2}(x_{32}, x_{\rho 2}) - \mathcal{S}L_{i_2}(x_{42}, x_{\rho 2}) \right) \right), \tag{63}
\end{aligned}$$

$$F_{(c),2}^{a,1} = \sum_{\rho=1}^3 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \ln x_{\rho\mu} - \ln^2 x_{\rho\mu} - \mathcal{S}L_{i_2}(x_{6\rho}, x_{7\rho}) \right),$$

$$F_{(c),2}^{b,1} = -\frac{1}{2} \sum_{\rho=1}^5 \frac{x_{\rho\mu}^2 x_{6\mu} + x_{\rho\mu} x_{6\mu}^2}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \ln^2(x_{\rho\mu} x_{6\mu}),$$

$$F_{(c),2}^{c,1} = -\frac{1}{2} \sum_{\rho=1}^5 \frac{x_{\rho\mu}^2 x_{7\mu} + x_{\rho\mu} x_{7\mu}^2}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \ln^2(x_{\rho\mu} x_{7\mu}),$$

$$F_{(c),2}^{d,1} = \frac{1}{2} \sum_{\rho=1}^4 \frac{x_{\rho\mu} x_{6\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \ln^2(x_{\rho\mu} x_{6\mu}),$$

$$F_{(c),2}^{e,1} = \frac{1}{2} \sum_{\rho=1}^4 \frac{x_{\rho\mu} x_{7\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \ln^2(x_{\rho\mu} x_{7\mu}),$$

$$F_{(c),2}^f = 0,$$

$$F_{(c),1}^{a,1} = -\sum_{\rho=1}^4 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \ln x_{\rho\mu} - \ln^2 x_{\rho\mu} - \mathcal{S}L_{i_2}(x_{6\rho}, x_{7\rho}) \right),$$

$$F_{(c),1}^{b,1} = -\frac{1}{2(4\pi)^4} \sum_{\rho=1}^5 \frac{x_{\rho\mu} x_{6\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \ln^2(x_{\rho\mu} x_{6\mu}),$$

$$F_{(c),1}^{c,1} = -\frac{1}{2} \sum_{\rho=1}^5 \frac{x_{\rho\mu} x_{7\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \ln^2(x_{\rho\mu} x_{7\mu}),$$

$$F_{(c),0} = \sum_{\rho=1}^5 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \ln x_{\rho\mu} - \ln^2 x_{\rho\mu} - \mathcal{S}L_{i_2}(x_{6\rho}, x_{7\rho}) \right), \tag{64}$$

$$\begin{aligned}
F_{(d),2}^{a,1} &= \frac{1}{(x_{1\mu} - x_{2\mu})(x_{6\mu} - x_{7\mu})} \left( x_{1\mu} \left( \mathcal{S}L_{i_2}(x_{51}, x_{61}) - \mathcal{S}L_{i_2}(x_{51}, x_{71}) \right) \right. \\
&\quad \left. - x_{2\mu} \left( \mathcal{S}L_{i_2}(x_{52}, x_{62}) - \mathcal{S}L_{i_2}(x_{52}, x_{72}) \right) \right),
\end{aligned}$$

$$\begin{aligned}
F_{(d),2}^{b,1} &= -\frac{1}{2(4\pi)^4} \sum_{\rho=1}^4 \frac{x_{\rho\mu} x_{5\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \ln^2(x_{\rho\mu} x_{5\mu}) , \\
F_{(d),2}^{c,1} &= \frac{1}{2} \frac{1}{x_{6\mu} - x_{7\mu}} \sum_{\rho=1}^4 \frac{1}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( x_{\rho\mu} x_{6\mu} (x_{\rho\mu} + x_{6\mu}) \ln^2(x_{\rho\mu} x_{6\mu}) \right. \\
&\quad \left. - x_{\rho\mu} x_{7\mu} (x_{\rho\mu} + x_{7\mu}) \ln^2(x_{\rho\mu} x_{7\mu}) \right) , \\
F_{(d),2}^{d,1} &= \sum_{\rho=1}^3 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \ln x_{\rho\mu} - \ln^2 x_{\rho\mu} - \mathcal{S}L_{i_2}(x_{5\rho}, x_{6\rho}) \right) , \\
F_{(d),2}^{e,1} &= -\sum_{\rho=1}^3 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \frac{x_{6\mu} \ln^2(x_{6\mu} x_{\rho\mu}) - x_{7\mu} \ln^2(x_{7\mu} x_{\rho\mu})}{2(x_{6\mu} - x_{7\mu})} , \\
F_{(d),2}^{f,1} &= -\frac{1}{2} \sum_{\rho=1}^3 \frac{x_{6\mu} x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \ln^2(x_{6\mu} x_{\rho\mu}) , \\
F_{(d),1}^{a,1} &= -\frac{1}{x_{6\mu} - x_{7\mu}} \sum_{\rho=1}^3 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \mathcal{S}L_{i_2}(x_{5\rho}, x_{6\rho}) - \mathcal{S}L_{i_2}(x_{5\rho}, x_{7\rho}) \right) , \\
F_{(d),1}^{b,1} &= -\sum_{\rho=1}^4 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \ln x_{\rho\mu} - \ln^2 x_{\rho\mu} \right) , \\
F_{(d),1}^{c,1} &= \frac{1}{2} \sum_{\rho=1}^4 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \frac{x_{6\mu} \ln^2(x_{6\mu} x_{\rho\mu}) - x_{7\mu} \ln^2(x_{7\mu} x_{\rho\mu})}{(x_{6\mu} - x_{7\mu})} , \\
F_{(d),0} &= \frac{1}{x_{6\mu} - x_{7\mu}} \sum_{\rho=1}^4 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \mathcal{S}L_{i_2}(x_{5\rho}, x_{6\rho}) - \mathcal{S}L_{i_2}(x_{5\rho}, x_{7\rho}) \right) , \tag{65}
\end{aligned}$$

$$\begin{aligned}
F_{(e),2}^{a,1} &= -\frac{1}{(x_{4\mu} - x_{5\mu})(x_{6\mu} - x_{7\mu})} \left( x_{6\mu} \left( \mathcal{S}L_{i_2}(x_{16}, x_{46}) - \mathcal{S}L_{i_2}(x_{16}, x_{56}) \right) \right. \\
&\quad \left. - x_{7\mu} \left( \mathcal{S}L_{i_2}(x_{17}, x_{47}) - \mathcal{S}L_{i_2}(x_{17}, x_{57}) \right) \right) , \\
F_{(e),2}^{b,1} &= -\frac{1}{2} \sum_{\rho=1}^3 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \frac{x_{4\mu} \ln^2(x_{4\mu} x_{\rho\mu}) - x_{5\mu} \ln^2(x_{5\mu} x_{\rho\mu})}{(x_{4\mu} - x_{5\mu})} , \\
F_{(e),2}^{c,1} &= -\frac{1}{2} \sum_{\rho=1}^3 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \frac{x_{6\mu} \ln^2(x_{6\mu} x_{\rho\mu}) - x_{7\mu} \ln^2(x_{7\mu} x_{\rho\mu})}{(x_{6\mu} - x_{7\mu})} , \\
F_{(e),2}^{d,1} &= -\frac{1}{(x_{1\mu} - x_{2\mu})(x_{4\mu} - x_{5\mu})} \left( x_{1\mu} \left( \mathcal{S}L_{i_2}(x_{41}, x_{61}) - \mathcal{S}L_{i_2}(x_{51}, x_{61}) \right) \right.
\end{aligned}$$

$$\begin{aligned}
& -x_{2\mu} \left( \mathcal{S}L_{i_2}(x_{42}, x_{62}) - \mathcal{S}L_{i_2}(x_{52}, x_{62}) \right) \Big) , \\
F_{(e),2}^{e,1} &= -\frac{1}{(x_{1\mu} - x_{2\mu})(x_{6\mu} - x_{7\mu})} \left( x_{1\mu} \left( \mathcal{S}L_{i_2}(x_{41}, x_{61}) - \mathcal{S}L_{i_2}(x_{41}, x_{71}) \right) \right. \\
& \quad \left. - x_{2\mu} \left( \mathcal{S}L_{i_2}(x_{42}, x_{62}) - \mathcal{S}L_{i_2}(x_{42}, x_{72}) \right) \right) , \\
F_{(e),2}^{f,1} &= \sum_{\rho=1}^3 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \ln x_{\rho\mu} - \ln^2 x_{\rho\mu} - \mathcal{S}L_{i_2}(x_{4\rho}, x_{6\rho}) \right) , \\
F_{(e),1}^{a,1} &= -\frac{1}{(x_{1\mu} - x_{2\mu})(x_{4\mu} - x_{5\mu})(x_{6\mu} - x_{7\mu})} \left( x_{1\mu} \left( \mathcal{S}L_{i_2}(x_{41}, x_{61}) - \mathcal{S}L_{i_2}(x_{41}, x_{71}) \right) \right. \\
& \quad \left. - \mathcal{S}L_{i_2}(x_{51}, x_{61}) + \mathcal{S}L_{i_2}(x_{51}, x_{71}) \right) - x_{2\mu} \left( \mathcal{S}L_{i_2}(x_{42}, x_{62}) - \mathcal{S}L_{i_2}(x_{42}, x_{72}) \right. \\
& \quad \left. - \mathcal{S}L_{i_2}(x_{52}, x_{62}) + \mathcal{S}L_{i_2}(x_{52}, x_{72}) \right) \Big) , \\
F_{(e),1}^{b,1} &= -\frac{1}{(x_{4\mu} - x_{5\mu})} \sum_{\rho=1}^3 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \mathcal{S}L_{i_2}(x_{4\rho}, x_{6\rho}) - \mathcal{S}L_{i_2}(x_{5\rho}, x_{6\rho}) \right) , \\
F_{(e),1}^{c,1} &= -\frac{1}{(x_{6\mu} - x_{7\mu})} \sum_{\rho=1}^3 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \mathcal{S}L_{i_2}(x_{4\rho}, x_{6\rho}) - \mathcal{S}L_{i_2}(x_{4\rho}, x_{7\rho}) \right) , \\
F_{(e),0} &= -\frac{1}{(x_{4\mu} - x_{5\mu})(x_{6\mu} - x_{7\mu})} \sum_{\rho=1}^3 \frac{x_{\rho\mu}}{\prod_{\sigma \neq \rho} (x_{\sigma\mu} - x_{\rho\mu})} \left( \mathcal{S}L_{i_2}(x_{4\rho}, x_{6\rho}) - \mathcal{S}L_{i_2}(x_{4\rho}, x_{7\rho}) \right. \\
& \quad \left. - \mathcal{S}L_{i_2}(x_{5\rho}, x_{6\rho}) + \mathcal{S}L_{i_2}(x_{5\rho}, x_{7\rho}) \right) . \tag{66}
\end{aligned}$$

The reduced formulae are similar to  $I_{(i)}$ , but with replacements of  $m_i^2 \rightarrow x_{i\mu}$ ,  $I_{(i)} \rightarrow F_{(i)}$ .

## C The expressions of $\mathcal{S}L_{i_2}(a, b)$ , $\mathcal{A}(a, b)$ , $\mathcal{R}_{i_2}(a)$ and $\Upsilon(a)$

We use the new symbols  $y_a, y_b$  to represent the two roots of equation  $a(1-x) + bx - x(1-x) = 0$ , and the functions  $\mathcal{S}L_{i_2}(a, b)$  can be written as

$$\begin{aligned}
\mathcal{S}L_{i_2}(a, b) &= 14 + \frac{(b-1)\ln^2 b}{2(b-a)} + \frac{(a-1)\ln^2 a}{2(a-b)} + \frac{1}{y_a - y_b} \left( \left( 2y_a + (b-a-1) \right) L_{i_2}\left(\frac{1}{1-y_a}\right) \right. \\
& \quad \left. - \left( 2y_b + (b-a-1) \right) L_{i_2}\left(\frac{1}{1-y_b}\right) - \left( \ln b \ln(1-y_b) - y_a \left( \ln(a + (b-a)y_a) \ln \frac{y_a-1}{y_a} \right. \right. \right. \\
& \quad \left. \left. + L_{i_2}\left(\frac{(a-b)y_a}{(a-b)y_a-a}\right) - L_{i_2}\left(\frac{(a-b)(y_a-1)}{(a-b)y_a-a}\right) \right) - \left( (2y_a-1) \ln \frac{y_a-1}{y_a} - y_a L_{i_2}\left(\frac{1}{y_a}\right) \right. \right. \\
& \quad \left. \left. + (y_a-1) L_{i_2}\left(\frac{1}{1-y_a}\right) - \ln(y_a(y_a-1)) \right) + (y_a \rightarrow y_b) \right) . \tag{67}
\end{aligned}$$

The expression of  $\mathcal{A}(a, b)$  is

$$\begin{aligned}\mathcal{A}(a, b) = & \frac{1}{2(b-a)} \left( b(\ln b + 1)^2 - a(\ln a + 1)^2 \right) - \frac{2b}{a-b} \ln a - \frac{2a}{b-a} \ln b \\ & - \frac{a}{b-a} \left( \ln \frac{b}{a} \ln \frac{a-b}{a} + L_{i_2}\left(\frac{b}{a}\right) \right) - \frac{b}{a-b} \left( \ln \frac{a}{b} \ln \frac{b-a}{b} + L_{i_2}\left(\frac{a}{b}\right) \right).\end{aligned}\quad (68)$$

It is easy to note that when  $a = b$ ,  $\mathcal{A}(a, b) = 2 \ln a + \frac{1}{2} \ln^2 a$ . The definition of function  $\Upsilon(a)$  is

$$\Upsilon(a) = - \left( \ln a \ln(1-a) + L_{i_2}(a) \right) + a \left( \ln a \ln(1 - \frac{1}{a}) - L_{i_2}(\frac{1}{a}) \right). \quad (69)$$

The expression of  $\mathcal{R}_{i_2}$  is written as

$$\mathcal{R}_{i_2}(a) = \ln a \ln(1-a) + L_{i_2}(a) - \frac{1}{2} \ln^2 a - \frac{\ln^2 a}{1-a}. \quad (70)$$

## D The coefficients at Next-to-leading Order

### D.1 The gluon corrections

We present here the gluon corrections  $L_{i,j}^\alpha$  ( $\alpha = 1, \dots, 8$ ) as follows

$$L_{i,j}^\alpha = WW_\alpha + 2WH_\alpha + HH_\alpha + cc_\alpha \quad (71)$$

with

$$\begin{aligned}WW_1 = & \frac{1}{3(-1+x_{iW})^3(x_{iW}-x_{jW})^2(-1+x_{jW})^2} \left( \left( 2x_{iW}(4-27x_{iW}+19x_{iW}^2-5x_{iW}^4+9x_{iW}^4) \right. \right. \\ & -2x_{jW}(4-30x_{iW}-39x_{iW}^2+17x_{iW}^3+39x_{iW}^4+x_{iW}^5) -2x_{jW}^2(3+79x_{iW}+49x_{iW}^2-91x_{iW}^3-40x_{iW}^4) \\ & +2x_{jW}^3(21+101x_{iW}-37x_{iW}^2-89x_{iW}^3+4x_{iW}^4) -4x_{jW}^4(15+20x_{iW}-32x_{iW}^2+2x_{iW}^3) \\ & \left. +32x_{jW}^5(1-x_{iW}) \right) \left( L_{i_2}\left(\frac{x_{jW}}{x_{iW}}\right) + \ln \frac{x_{jW}}{x_{iW}} \ln(1-\frac{x_{jW}}{x_{iW}}) \right) \\ & + \left( -2x_{iW}(4+3x_{iW}-41x_{iW}^2+65x_{iW}^3-39x_{iW}^4+8x_{iW}^5) +2x_{jW}(4+30x_{iW}-155x_{iW}^2+265x_{iW}^3 \right. \\ & -221x_{iW}^4+93x_{iW}^5-16x_{iW}^6) -2x_{jW}^2(27-99x_{iW}+171x_{iW}^2-181x_{iW}^3+110x_{iW}^4-28x_{iW}^5) \\ & \left. +2x_{jW}^3(15-29x_{iW}+x_{iW}^2+25x_{iW}^3-12x_{iW}^4) \right) \left( L_{i_2}\left(\frac{x_{iW}}{x_{jW}}\right) + \ln \frac{x_{iW}}{x_{jW}} \ln(1-\frac{x_{iW}}{x_{jW}}) \right) \\ & + \left( 4x_{iW}^2(13-53x_{iW}+55x_{iW}^2-19x_{iW}^3+4x_{iW}^4) -4x_{iW}x_{jW}(20-77x_{iW}+77x_{jW}^2-59x_{iW}^3+47x_{iW}^4-8x_{iW}^5) \right. \\ & \left. +4x_{jW}^2(7-21x_{iW}+29x_{iW}^2-85x_{iW}^3+84x_{iW}^4-14x_{iW}^5) -4x_{jW}^3(3+7x_{iW}-45x_{iW}^2+41x_{iW}^3 \right.\end{aligned}$$

$$\begin{aligned}
& -6x_{iw}^4) \Big) (L_{i_2}(x_{iw}) + \ln x_{iw} \ln(1 - x_{iw})) \\
& + \Big( 4x_{iw}^2(3 + 10x_{iw} - 13x_{iw}^2) - 4x_{iw}x_{jw}(12 + 10x_{iw} + 16x_{iw}^2 - 38x_{iw}^3) + 4x_{jw}^2(9 + 16x_{iw} + 46x_{iw}^2 \\
& - 48x_{iw}^3 - 23x_{iw}^4) - 8x_{jw}^3(8 + 16x_{iw} + x_{iw}^2 - 26x_{iw}^3 + x_{iw}^4) + 4x_{jw}^4(15 + 20x_{iw} - 37x_{iw}^2 + 2x_{iw}^3) \\
& - 32x_{jw}^5(1 - x_{iw}) \Big) (L_{i_2}(x_{jw}) + \ln x_{jw} \ln(1 - x_{jw})) \\
& + \Big( 8x_{iw}(3 + x_{iw} - 104x_{iw}^2 + 98x_{iw}^3 - 46x_{iw}^4) + 8x_{jw}(3 - 29x_{iw} + 209x_{iw}^2 - 110x_{iw}^2 + 25x_{iw}^4 + 46x_{iw}^5) \\
& - 8x_{jw}^2(2 + 51x_{iw} + 69x_{iw}^2 - 86x_{iw}^3 + 108x_{iw}^4) + 8x_{jw}^3(30 - 41x_{iw} + 3x_{iw}^2 + 50x_{iw}^3 + 6x_{iw}^4) \\
& - 32x_{jw}^4(4 - 13x_{iw} + 9x_{iw}^2) \Big) + \Big( 8x_{iw}^2(47 - 80x_{iw} - 42x_{iw}^2 - x_{iw}^3) + 8x_{iw}x_{jw}(-47 + 13x_{iw} + 191x_{iw}^2 \\
& + 70x_{iw}^3 + x_{iw}^4) + 8x_{iw}x_{jw}^2(105 - 211x_{iw} - 94x_{iw}^2 - 26x_{iw}^3) - 48x_{iw}x_{jw}^4(1 - x_{iw}) \Big) \ln x_{iw} \\
& + \Big( x_{iw}(4 + 175x_{iw} + 38x_{iw}^2 + 57x_{iw}^3 - 34x_{iw}^4) + 2x_{jw}(2 - 179x_{iw} - 116x_{iw}^2 - 69x_{iw}^3 - 31x_{iw}^4 + 33x_{iw}^5) \\
& - x_{jw}^2(5 - 751x_{iw} + 88x_{iw}^2 - 125x_{iw}^3 + 31x_{iw}^4 + 32x_{iw}^5) - x_{jw}^3(13 + 453x_{iw} - 219x_{iw}^2 + 29x_{iw}^3 - 36x_{iw}^4) \\
& + 2x_{jw}^4(15 + 20x_{iw} - 37x_{iw}^2 + 2x_{iw}^3) - 16x_{jw}^5(1 - x_{iw}) \Big) \ln^2 x_{iw} \\
& + \Big( -16x_{iw}^2(4 - 9x_{iw} + 6x_{iw}^2 - x_{iw}^3) + 8x_{iw}x_{jw}(13 - 27x_{iw} + 15x_{iw}^2 - x_{iw}^3) - 8x_{jw}^2(5 + 29x_{iw} \\
& - 117x_{iw}^2 + 127x_{iw}^3 - 44x_{iw}^4) - 96x_{iw}x_{jw}^3(1 - x_{iw})^2 - 48x_{jw}^4(1 - x_{iw})^2 \Big) \ln x_{jw} \\
& + \Big( -2x_{iw}(4 + 15x_{iw} - 41x_{iw}^2 + 53x_{iw}^3 - 39x_{iw}^4 + 8x_{iw}^5) + 2x_{jw}(-4 + 110x_{iw} - 189x_{iw}^2 + 145x_{iw}^3 \\
& - 79x_{iw}^4 + 33x_{iw}^5 - 16x_{iw}^6) + 2x_{jw}^2(5 - 137x_{iw} + 119x_{iw}^2 + 101x_{iw}^3 - 148x_{iw}^4 + 60x_{iw}^5) \\
& + 2x_{jw}^3(13 + 123x_{iw} - 237x_{iw}^2 + 149x_{iw}^3 - 48x_{iw}^4) - 4x_{jw}^4(15 - 17x_{iw} + 2x_{iw}^2) \\
& + 32x_{jw}^5(1 - x_{iw}) \Big) \ln x_{iw} \ln x_{jw} \\
& + \Big( x_{iw}(4 + 3x_{iw} - 41x_{iw}^2 + 65x_{iw}^3 - 39x_{iw}^4) + 8x_{iw}^5) + x_{jw}(4 + 18x_{iw} + 39x_{iw}^2 - 285x_{iw}^2 + 377x_{iw}^3 \\
& - 169x_{iw}^4 + 16x_{iw}^5) + x_{jw}^2(-33 + 12x_{iw} + 230x_{iw}^2 - 424x_{iw}^3 + 275x_{iw}^4 - 60x_{iw}^5) \\
& + x_{jw}^3(-10 - 26x_{iw} + 134x_{iw}^2 - 150x_{iw}^3 + 52x_{iw}^4) \Big) \ln^2 x_{jw} + (x_{iw} \leftrightarrow x_{jw}) \Big) , \tag{72}
\end{aligned}$$

$$WH_1 = \frac{(Z_H^{2k})^2}{\sin^2 \beta} x_{iw} x_{jw} \left( \left( \frac{16}{(-1 + x_{iw})(x_{H_k^-} - x_{iw})} - \frac{16}{(-1 + x_{iw})(x_{H_k^-} - x_{jw})} \right) \left( L_{i_2} \left( \frac{x_{jw}}{x_{iw}} \right) \right. \right.$$

$$\begin{aligned}
& + \ln \frac{x_{jw}}{x_{iw}} \ln(1 - \frac{x_{jw}}{x_{iw}})) \\
& + \left( \frac{32}{3(x_{iw} - 1)(x_{jw} - 1)} + \frac{32}{3(x_{iw} - 1)^2(x_{jw} - 1)} - \frac{32x_{H_k^-w}}{3(x_{iw} - 1)^2(x_{jw} - 1)(x_{H_k^-w} - 1)} \right. \\
& + \frac{32x_{H_k^-w}}{3(x_{iw} - 1)(x_{jw} - 1)(x_{H_k^-w} - 1)} + \frac{4}{3(x_{iw} - 1)(x_{jw} - 1)(x_{iw} - x_{jw})} \Big) \left( L_{i2}(x_{iw}) + \ln x_{iw} \ln(1 - x_{iw}) \right) \\
& - \left( \frac{16}{(-1 + x_{iw})(x_{H_k^-w} - 1)} + \frac{16}{(x_{iw} - 1)(x_{H_k^-w} - x_{jw})} + \frac{4}{3(x_{iw} - 1)(x_{jw} - 1)(x_{iw} - x_{jw})} \right) \\
& \left( L_{i2}(x_{jw}) + \ln x_{jw} \ln(1 - x_{jw}) \right) - \frac{16}{3(x_{H_k^-w} - 1)(x_{H_k^-w} - x_{iw})} \left( L_{i2}(\frac{x_{jw}}{x_{H_k^-w}}) + \ln \frac{x_{jw}}{x_{H_k^-w}} \ln(1 - \frac{x_{jw}}{x_{H_k^-w}}) \right) \\
& + \frac{32x_{H_k^-w}^2(1 + x_{H_k^-w} - x_{iw})}{3(-1 + x_{H_k^-w})(x_{H_k^-w} - x_{iw})^2(x_{H_k^-w} - x_{jw})} \left( L_{i2}(\frac{x_{iw}}{x_{H_k^-w}}) + \ln \frac{x_{iw}}{x_{H_k^-w}} \ln(1 - \frac{x_{iw}}{x_{H_k^-w}}) \right) \\
& - \left( \frac{4x_{H_k^-w}^2(1 + x_{H_k^-w})}{3(x_{H_k^-w} - x_{iw})(x_{iw} - 1)(x_{H_k^-w} - x_{jw})(x_{iw} - x_{jw})} + \frac{4x_{H_k^-w}}{3(x_{H_k^-w} - x_{iw})(x_{H_k^-w} - x_{jw})(x_{iw} - x_{jw})} \right. \\
& + \frac{4}{3(x_{iw} - 1)(x_{iw} - x_{jw})} - \frac{16}{3(x_{iw} - 1)(x_{H_k^-w} - x_{jw})} \Big) \left( L_{i2}(\frac{x_{H_k^-w}}{x_{iw}}) + \ln \frac{x_{H_k^-w}}{x_{iw}} \ln(1 - \frac{x_{H_k^-w}}{x_{iw}}) \right) \\
& + \left( \frac{4x_{H_k^-w}^2(1 + x_{H_k^-w})}{3(x_{H_k^-w} - x_{iw})(x_{jw} - 1)(x_{H_k^-w} - x_{jw})(x_{iw} - x_{jw})} + \frac{4x_{H_k^-w}}{3(x_{H_k^-w} - x_{iw})(x_{H_k^-w} - x_{jw})(x_{iw} - x_{jw})} \right. \\
& - \frac{4}{3(x_{iw} - 1)(x_{iw} - x_{jw})} \Big) \left( L_{i2}(\frac{x_{H_k^-w}}{x_{jw}}) + \ln \frac{x_{H_k^-w}}{x_{jw}} \ln(1 - \frac{x_{H_k^-w}}{x_{jw}}) \right) \\
& + \left( \frac{4x_{H_k^-w}(1 - x_{H_k^-w}^2)}{3(x_{H_k^-w} - x_{iw})(x_{H_k^-w} - x_{jw})(x_{iw} - 1)(x_{jw} - 1)} - \frac{4}{3(x_{iw} - 1)(x_{jw} - 1)} \right. \\
& + \frac{16}{3(x_{iw} - 1)(x_{H_k^-w} - x_{jw})} \Big) \left( L_{i2}(x_{H_k^-w}) + \ln x_{H_k^-w} \ln(1 - x_{H_k^-w}) \right) \\
& - \frac{128 + 32 \ln x_{iw} + 30 \ln^2 x_{iw} + 4 \ln x_{H_k^-w} \ln x_{iw}}{3(x_{iw} - 1)(x_{iw} - x_{jw})} + \frac{8 \ln^2(\frac{x_{H_k^-w}}{x_{jw}})}{3(x_{H_k^-w} - 1)(x_{H_k^-w} - x_{iw})} \\
& + \frac{2 \ln^2 x_{H_k^-w} - 16 \ln^2 x_{iw}}{3(x_{iw} - 1)(x_{jw} - 1)} - \frac{16(\ln^2 x_{H_k^-w} - \ln x_{iw} \ln x_{H_k^-w} + \ln x_{iw} \ln x_{jw})}{3(x_{iw} - 1)(x_{H_k^-w} - x_{jw})} \\
& + \frac{8 \ln^2 \frac{x_{iw}}{x_{jw}}}{3(x_{iw} - 1)(x_{H_k^-w} - 1)} + \frac{4 \ln x_{H_k^-w} \ln x_{jw} - 2 \ln^2 x_{jw}}{3(x_{iw} - x_{jw})(x_{jw} - 1)} + \frac{32 \ln^2 x_{iw}}{3(x_{iw} - 1)^2(x_{jw} - 1)} \\
& + \frac{8 \ln^2 x_{jw}}{(x_{iw} - 1)(x_{iw} - x_{H_k^-w})} + \frac{2x_{H_k^-w}^2(1 + x_{H_k^-w}) \ln^2(\frac{x_{H_k^-w}}{x_{iw}})}{3(x_{H_k^-w} - x_{iw})(x_{iw} - 1)(x_{H_k^-w} - x_{jw})(x_{iw} - x_{jw})}
\end{aligned}$$

$$\begin{aligned}
& + \frac{x_{H_k^- w}(2x_{H_k^- w} + 1)(x_{H_k^- w} - 1) \ln^2 x_{H_k^- w}}{3(x_{H_k^- w} - x_{i w})(x_{i w} - 1)(x_{H_k^- w} - x_{j w})(x_{j w} - 1)} - \frac{4x_{j w} \ln^2 \frac{x_{j w}}{x_{H_k^- w}}}{3(x_{H_k^- w} - x_{j w})(x_{j w} - 1)(x_{j w} - x_{i w})} \\
& - \frac{2x_{H_k^- w}^2(1 + x_{H_k^- w}) \ln^2 \frac{x_{j w}}{x_{H_k^- w}}}{3(x_{H_k^- w} - x_{i w})(x_{i w} - x_{j w})(x_{H_k^- w} - x_{j w})(x_{j w} - 1)} - \frac{4(\ln^2 x_{H_k^- w} - 4x_{H_k^- w} \ln x_{i w} + 4 \ln^2 x_{j w})}{3(x_{H_k^- w} - 1)(x_{i w} - 1)(x_{j w} - 1)} \\
& + \frac{32x_{H_k^- w}(\ln^2 x_{i w} + 3 \ln x_{i w} + 4) - 4x_{i w} \ln^2 \frac{x_{H_k^- w}}{x_{i w}} - 16x_{i w} \ln^2 \frac{x_{j w}}{x_{i w}}}{3(x_{H_k^- w} - x_{i w})(x_{i w} - 1)(x_{i w} - x_{j w})} \\
& - \frac{2x_{H_k^- w}(2 \ln x_{H_k^- w} - \ln(x_{i w} x_{j w})) \ln \frac{x_{i w}}{x_{j w}}}{3(x_{H_k^- w} - x_{i w})(x_{H_k^- w} - x_{j w})(x_{i w} - x_{j w})} - \frac{16 \ln^2 x_{j w}}{3(x_{i w} - 1)(x_{j w} - 1)(x_{j w} - x_{H_k^- w})} \\
& - \frac{16x_{H_k^- w}^2 \ln^2 \frac{x_{H_k^- w}}{x_{i w}} + 16x_{H_k^- w}(7 \ln x_{H_k^- w} + 2 \ln x_{H_k^- w} \ln x_{j w} - \ln^2 x_{j w})}{3(x_{H_k^- w} - 1)(x_{H_k^- w} - x_{i w})(x_{H_k^- w} - x_{j w})} \\
& + \frac{16x_{i w} \ln^2 \frac{x_{i w}}{x_{j w}}}{3(x_{i w} - 1)(x_{H_k^- w} - x_{j w})(x_{i w} - x_{j w})} - \frac{32x_{H_k^- w} \ln^2 x_{i w}}{3(x_{i w} - 1)^2(x_{H_k^- w} - 1)(x_{j w} - 1)} \\
& + \frac{16x_{i w} \ln^2 \frac{x_{H_k^- w}}{x_{i w}}}{3(x_{H_k^- w} - x_{i w})(x_{i w} - 1)(x_{H_k^- w} - x_{j w})} + \frac{16 \ln^2 x_{H_k^- w}}{3(x_{H_k^- w} - 1)(x_{i w} - 1)(x_{H_k^- w} - x_{j w})} \\
& - \frac{32x_{H_k^- w}^2(\ln x_{H_k^- w} + x_{H_k^- w} \ln^2 x_{H_k^- w} + x_{H_k^- w} \ln^2 \frac{x_{H_k^- w}}{x_{i w}})}{3(x_{H_k^- w} - 1)(x_{H_k^- w} - x_{i w})^2(x_{H_k^- w} - x_{j w})} + (x_{i w} \leftrightarrow x_{j w}) \\
& + (\mathcal{Z}_H^{1k})^2 h_b h_d \sqrt{x_{i w} x_{j w}} \left( \left( \frac{8}{3(x_{j w} - x_{H_k^- w})(x_{j w} - x_{i w})} - \frac{8(1 - x_{j w})}{3(x_{i w} - 1)(x_{i w} - x_{j w})(x_{j w} - x_{H_k^- w})} \right) \right. \\
& \left. \left( L_{i_2} \left( \frac{x_{j w}}{x_{i w}} \right) - \ln \frac{x_{j w}}{x_{i w}} \ln \left( 1 - \frac{x_{i w}}{x_{j w}} \right) \right) + \frac{8(1 - x_{j w})}{3(x_{i w} - 1)(x_{j w} - 1)(x_{H_k^- w} - x_{j w})} \left( L_{i_2}(x_{j w}) + \ln x_{j w} \ln \left( 1 - x_{j w} \right) \right) \right. \\
& + \left( \frac{8(1 - x_{j w})}{3(x_{i w} - 1)(x_{i w} - x_{j w})(x_{j w} - x_{H_k^- w})} + \frac{8}{3(x_{i w} - x_{j w})(x_{j w} - x_{H_k^- w})} \right. \\
& + \left. \frac{8}{3(x_{i w} - 1)(x_{i w} - x_{j w})} \right) \left( L_{i_2} \left( \frac{x_{H_k^- w}}{x_{i w}} \right) + \ln \frac{x_{H_k^- w}}{x_{i w}} \ln \left( 1 - \frac{x_{H_k^- w}}{x_{i w}} \right) \right) \\
& - \left( \frac{8(1 - x_{j w})}{3(x_{j w} - 1)(x_{i w} - x_{j w})(x_{j w} - x_{H_k^- w})} + \frac{8}{3(x_{i w} - x_{j w})(x_{j w} - x_{H_k^- w})} \right. \\
& + \left. \frac{8}{3(x_{i w} - 1)(x_{i w} - x_{j w})} \right) \left( L_{i_2} \left( \frac{x_{H_k^- w}}{x_{j w}} \right) + \ln \frac{x_{H_k^- w}}{x_{j w}} \ln \left( 1 - \frac{x_{H_k^- w}}{x_{j w}} \right) \right) \\
& + \left( \frac{8(x_{j w} - 1)}{3(x_{i w} - 1)(x_{j w} - 1)(x_{H_k^- w} - x_{j w})} + \frac{8}{3(x_{i w} - 1)(x_{j w} - 1)} \right) \left( L_{i_2}(x_{H_k^- w}) + \ln x_{H_k^- w} \ln \left( 1 - x_{H_k^- w} \right) \right)
\end{aligned}$$



$$\begin{aligned}
& + \frac{8x_{i\bar{w}}(x_{j\bar{w}} - 1) \ln^2 \frac{x_{H_k^- \bar{w}}}{x_{i\bar{w}}}}{3(x_{H_k^- \bar{w}} - x_{i\bar{w}})(x_{i\bar{w}} - 1)(x_{H_k^- \bar{w}} - x_{j\bar{w}})(x_{i\bar{w}} - x_{j\bar{w}})} + \frac{8(1 - x_{j\bar{w}}) \ln^2 x_{H_k^- \bar{w}}}{3(x_{H_k^- \bar{w}} - 1)(x_{i\bar{w}} - 1)(x_{j\bar{w}} - 1)(x_{j\bar{w}} - x_{H_k^- \bar{w}})} \\
& - \frac{8x_{j\bar{w}}(1 - x_{j\bar{w}}) \ln^2 \frac{x_{H_k^- \bar{w}}}{x_{j\bar{w}}}}{3(x_{H_k^- \bar{w}} - x_{j\bar{w}})^2(x_{j\bar{w}} - 1)(x_{j\bar{w}} - x_{i\bar{w}})} + \frac{8(1 - x_{j\bar{w}})}{3(x_{H_k^- \bar{w}} - x_{j\bar{w}})(x_{i\bar{w}} - 1)(x_{j\bar{w}} - 1)^2} \\
& + \frac{8x_{i\bar{w}} \ln^2 \frac{x_{H_k^- \bar{w}}}{x_{i\bar{w}}}}{3(x_{H_k^- \bar{w}} - x_{i\bar{w}})(x_{i\bar{w}} - 1)(x_{i\bar{w}} - x_{j\bar{w}})} - \frac{4(1 + x_{j\bar{w}}) \ln^2 \frac{x_{H_k^- \bar{w}}}{x_{j\bar{w}}}}{3(x_{H_k^- \bar{w}} - x_{j\bar{w}})(x_{i\bar{w}} - x_{j\bar{w}})(x_{j\bar{w}} - 1)} \\
& + \frac{8x_{i\bar{w}}(x_{j\bar{w}} - 1) \ln^2 \frac{x_{i\bar{w}}}{x_{j\bar{w}}}}{3(x_{H_k^- \bar{w}} - x_{j\bar{w}})(x_{i\bar{w}} - 1)(x_{i\bar{w}} - x_{j\bar{w}})^2} - \frac{8x_{i\bar{w}} \ln^2 \frac{x_{H_k^- \bar{w}}}{x_{i\bar{w}}}}{3(x_{H_k^- \bar{w}} - x_{i\bar{w}})(x_{H_k^- \bar{w}} - x_{j\bar{w}})(x_{i\bar{w}} - x_{j\bar{w}})} \\
& + \frac{8 \ln^2 x_{H_k^- \bar{w}}}{3(x_{H_k^- \bar{w}} - 1)(x_{i\bar{w}} - 1)(x_{j\bar{w}} - 1)} - \frac{4(x_{j\bar{w}} - 1) \ln(\frac{x_{H_k^- \bar{w}}}{x_{j\bar{w}}}) \ln(\frac{x_{H_k^- \bar{w}} x_{j\bar{w}}}{x_{i\bar{w}}^2})}{3(x_{i\bar{w}} - 1)(x_{H_k^- \bar{w}} - x_{j\bar{w}})(x_{i\bar{w}} - x_{j\bar{w}})} \\
& - \frac{4(\ln^2 x_{H_k^- \bar{w}} - \ln^2 x_{j\bar{w}})}{3(x_{i\bar{w}} - 1)(x_{H_k^- \bar{w}} - x_{j\bar{w}})} + \frac{8x_{j\bar{w}} \ln^2(\frac{x_{H_k^- \bar{w}}}{x_{j\bar{w}}})}{3(x_{H_k^- \bar{w}} - x_{j\bar{w}})^2(x_{i\bar{w}} - x_{j\bar{w}})} + \frac{4 \ln^2 x_{H_k^- \bar{w}}}{3(x_{i\bar{w}} - 1)(1 - x_{j\bar{w}})} \\
& - \frac{4 \ln^2 \frac{x_{H_k^- \bar{w}}}{x_{i\bar{w}}}}{3(x_{i\bar{w}} - 1)(x_{i\bar{w}} - x_{j\bar{w}})} + \frac{8(\ln x_{H_k^- \bar{w}} \ln x_{i\bar{w}} - \ln x_{H_k^- \bar{w}} \ln x_{j\bar{w}} - \ln x_{i\bar{w}} \ln x_{j\bar{w}} + \ln^2 x_{j\bar{w}})}{3(x_{i\bar{w}} - x_{j\bar{w}})(x_{j\bar{w}} - x_{H_k^- \bar{w}})} \\
& + \frac{8x_{i\bar{w}}(\ln^2 \frac{x_{H_k^- \bar{w}}}{x_{i\bar{w}}})}{3(x_{i\bar{w}} - x_{j\bar{w}})^2(x_{j\bar{w}} - x_{H_k^- \bar{w}})} - \frac{4 \ln^2(\frac{x_{H_k^- \bar{w}}}{x_{j\bar{w}}})}{3(x_{j\bar{w}} - 1)(x_{j\bar{w}} - x_{i\bar{w}})} + (x_{i\bar{w}} \leftrightarrow x_{j\bar{w}}) \Big), \tag{73}
\end{aligned}$$

$$\begin{aligned}
WH_2 &= \frac{1}{2} h_b h_d (Z_H^{1k})^2 \left( \left( \frac{x_{j\bar{w}}((1 + 2x_{H_k^- \bar{w}})^2 + 2(x_{i\bar{w}} + x_{j\bar{w}})(1 + x_{H_k^- \bar{w}}) - 8x_{i\bar{w}}x_{j\bar{w}})}{6(-1 + x_{i\bar{w}})(x_{H_k^- \bar{w}} - x_{j\bar{w}})(x_{i\bar{w}} - x_{j\bar{w}})^2} - \frac{14}{3(1 - x_{i\bar{w}})} \right. \right. \\
& + \frac{2}{3(x_{i\bar{w}} - x_{j\bar{w}})} + \frac{2(1 + x_{H_k^- \bar{w}})}{3(-1 + x_{i\bar{w}})(x_{H_k^- \bar{w}} - x_{j\bar{w}})} - \frac{16(x_{H_k^- \bar{w}} + x_{i\bar{w}})}{3(-1 + x_{i\bar{w}})(x_{H_k^- \bar{w}} - x_{i\bar{w}})} \\
& + \frac{2 + x_{H_k^- \bar{w}}}{3(x_{i\bar{w}} - 1)(x_{i\bar{w}} - x_{j\bar{w}})} - \frac{4x_{i\bar{w}}^2 - x_{i\bar{w}}x_{j\bar{w}}}{6(x_{i\bar{w}} - 1)(x_{i\bar{w}} - x_{j\bar{w}})^2} + \frac{x_{i\bar{w}}(2x_{i\bar{w}} - x_{j\bar{w}}) - x_{j\bar{w}}(1 + x_{H_k^- \bar{w}})^2}{2(x_{H_k^- \bar{w}} - x_{j\bar{w}})(x_{i\bar{w}} - x_{j\bar{w}})^2} \Big) \\
& \left( L_{i_2}(\frac{x_{j\bar{w}}}{x_{i\bar{w}}}) + \ln \frac{x_{j\bar{w}}}{x_{i\bar{w}}} \ln(1 - \frac{x_{j\bar{w}}}{x_{i\bar{w}}}) \right) \\
& + \left( \frac{x_{i\bar{w}}(8x_{i\bar{w}}x_{j\bar{w}} - 2(x_{i\bar{w}} + x_{j\bar{w}})(1 + x_{H_k^- \bar{w}}) - (1 + x_{H_k^- \bar{w}})^2)}{6(x_{j\bar{w}} - 1)(x_{i\bar{w}} - x_{j\bar{w}})(x_{H_k^- \bar{w}} - x_{i\bar{w}})} - \frac{8(x_{i\bar{w}}^2 + 2x_{i\bar{w}}^2x_{j\bar{w}} - 2x_{H_k^- \bar{w}}x_{j\bar{w}})}{3(x_{j\bar{w}} - 1)(x_{i\bar{w}} - x_{j\bar{w}})(x_{H_k^- \bar{w}} - x_{j\bar{w}})} \right. \\
& + \frac{x_{i\bar{w}}(3(1 + x_{H_k^- \bar{w}})^2(x_{i\bar{w}} - 1) + 16x_{H_k^- \bar{w}})}{6(x_{j\bar{w}} - 1)(x_{i\bar{w}} - x_{j\bar{w}})^2(x_{H_k^- \bar{w}} - x_{i\bar{w}})} - \frac{8x_{H_k^- \bar{w}}(x_{H_k^- \bar{w}}x_{j\bar{w}} - 2x_{i\bar{w}}(1 + x_{j\bar{w}}))}{3(x_{j\bar{w}} - 1)(x_{i\bar{w}} - x_{j\bar{w}})^2(x_{H_k^- \bar{w}} - x_{j\bar{w}})} \Big)
\end{aligned}$$

$$\begin{aligned}
& -\frac{32x_{iw}x_{H_k^-w} + x_{iw} + x_{jw}}{6(x_{iw} - x_{jw})(x_{H_k^-w} - x_{jw})} + \frac{1 + 2x_{H_k^-w} - x_{iw}}{6(x_{iw} - x_{jw})(x_{H_k^-w} - x_{iw})} - \frac{14 + 15x_{H_k^-w} - 96x_{iw} - 31x_{jw}}{6(x_{iw} - 1)(x_{iw} - x_{jw})} \\
& -\frac{10 - 32x_{iw}}{3(x_{iw} - x_{jw})} - \frac{2 + x_{H_k^-w} - x_{iw}}{2(x_{jw} - 1)(x_{iw} - x_{jw})} + \frac{1}{2(x_{H_k^-w} - x_{jw})} \Bigg) \Big( L_{i_2}\left(\frac{x_{iw}}{x_{jw}}\right) + \ln \frac{x_{iw}}{x_{jw}} \ln\left(1 - \frac{x_{iw}}{x_{jw}}\right) \Big) \\
& + \Big( \frac{x_{iw}((x_{iw} + x_{jw})(1 + x_{H_k^-w}) - 4x_{iw}x_{jw} - (1 + x_{H_k^-w})^2)}{6(x_{jw} - 1)(x_{iw} - x_{jw})(x_{H_k^-w} - x_{iw})} + \frac{x_{iw} - 2 - 4x_{H_k^-w}}{2(x_{iw} - x_{jw})(x_{jw} - 1)} \\
& + \frac{2 - x_{H_k^-w}^2(x_{iw} - 6) - 3x_{iw} - 16x_{iw}^2 - x_{H_k^-w}(97x_{iw} - 33)}{6(x_{jw} - 1)(x_{iw} - 1)(x_{H_k^-w} - 1)} - \frac{x_{H_k^-w}}{2(x_{jw} - 1)(x_{iw} - 1)(x_{iw} - x_{jw})} \\
& + \frac{15 + 97x_{iw} - 16x_{H_k^-w}}{6(x_{jw} - 1)(x_{iw} - 1)} + \frac{32x_{iw}^2}{3(x_{jw} - 1)(x_{iw} - 1)^2} + \frac{1}{2(-1 + x_{jw})} \Bigg) \Big( L_{i_2}(x_{iw}) + \ln x_{iw} \ln(1 - x_{iw}) \Big) \\
& + \Big( \frac{x_{jw}((x_{iw} + x_{jw})(1 + x_{H_k^-w}) - 4x_{iw}x_{jw} - (1 + x_{H_k^-w})^2)}{3(x_{iw} - 1)(x_{iw} - x_{jw})(x_{H_k^-w} - x_{jw})} + \frac{3x_{iw} - 5x_{H_k^-w} - 8}{3(x_{iw} - 1)(x_{H_k^-w} - x_{jw})} - \frac{14}{3(x_{iw} - 1)} \\
& + \frac{2 + x_{H_k^-w} - x_{iw}}{6(x_{iw} - 1)(x_{iw} - x_{jw})} + \frac{16(1 + x_{H_k^-w})}{3(x_{H_k^-w} - 1)(x_{iw} - 1)} \Bigg) \Big( L_{i_2}(x_{jw}) + \ln x_{jw} \ln(1 - x_{jw}) \Big) \\
& + \Big( \frac{8(x_{H_k^-w}^2 - 6x_{H_k^-w}x_{iw} + x_{iw}^2)}{3(x_{H_k^-w} - 1)(x_{H_k^-w} - x_{iw})(x_{H_k^-w} - x_{jw})} + \frac{x_{H_k^-w} + x_{iw}}{3(x_{H_k^-w} - x_{iw})(x_{H_k^-w} - x_{jw})} \\
& - \frac{16x_{H_k^-w}(1 + x_{iw})}{3(x_{H_k^-w} - 1)(x_{H_k^-w} - x_{jw})} \Bigg) \Big( L_{i_2}\left(\frac{x_{iw}}{x_{H_k^-w}}\right) + \ln\left(\frac{x_{iw}}{x_{H_k^-w}}\right) \ln\left(1 - \frac{x_{iw}}{x_{H_k^-w}}\right) \Big) \\
& + \frac{32}{3(x_{H_k^-w} - 1)(x_{H_k^-w} - x_{iw})} \Big( L_{i_2}\left(\frac{x_{jw}}{x_{H_k^-w}}\right) + \ln\left(\frac{x_{jw}}{x_{H_k^-w}}\right) \ln\left(1 - \frac{x_{jw}}{x_{H_k^-w}}\right) \Big) \\
& + \Big( \frac{x_{H_k^-w}((1 + x_{H_k^-w})^2 - (1 + x_{H_k^-w})(x_{iw} + x_{jw}) + 4x_{iw}x_{jw})}{6(-1 + x_{iw})(x_{H_k^-w} - x_{jw})(x_{iw} - x_{jw})} - \frac{x_{iw}}{(x_{H_k^-w} - x_{iw})(x_{H_k^-w} - x_{jw})} \\
& + \frac{2(x_{H_k^-w} + 2x_{H_k^-w}^2 - x_{iw} + x_{iw}^2) - 9x_{H_k^-w}x_{iw}}{6(x_{H_k^-w} - x_{iw})(x_{H_k^-w} - x_{jw})(x_{iw} - x_{jw})} - \frac{2(1 + x_{H_k^-w}) + x_{iw}}{3(-1 + x_{iw})(x_{H_k^-w} - x_{jw})} \\
& + \frac{2(2 - x_{H_k^-w} - x_{iw})}{3(-1 + x_{iw})(x_{iw} - x_{jw})} - \frac{4}{3(x_{iw} - x_{jw})} \Bigg) \Big( L_{i_2}\left(\frac{x_{H_k^-w}}{x_{iw}}\right) + \ln \frac{x_{H_k^-w}}{x_{iw}} \ln\left(1 - \frac{x_{H_k^-w}}{x_{iw}}\right) \Big) \\
& + \Big( \frac{x_{H_k^-w}((1 + x_{H_k^-w})(x_{iw} + x_{jw}) - (1 + x_{H_k^-w})^2 - 4x_{iw}x_{jw})}{3(-1 + x_{iw})(x_{H_k^-w} - x_{jw})(x_{iw} - x_{jw})} - \frac{2(1 + 2x_{H_k^-w} - x_{iw})}{(x_{H_k^-w} - x_{iw})(x_{iw} - x_{jw})} \\
& + \frac{2(2 - x_{H_k^-w} - x_{jw})}{(x_{jw} - 1)(x_{iw} - x_{jw})} - \frac{4}{3(x_{iw} - x_{jw})} \Bigg) \Big( L_{i_2}\left(\frac{x_{H_k^-w}}{x_{jw}}\right) + \ln \frac{x_{H_k^-w}}{x_{jw}} \ln\left(1 - \frac{x_{H_k^-w}}{x_{jw}}\right) \Big)
\end{aligned}$$

$$\begin{aligned}
& + \left( \frac{x_{H_k^- w} (x_{H_k^- w} - 1) ((1 + x_{H_k^- w})^2 - (x_{iw} + x_{jw} (1 + x_{H_k^- w}) + 4x_{iw} x_{jw}))}{2(x_{H_k^- w} - x_{iw})(x_{H_k^- w} - x_{jw})(-1 + x_{iw})(-1 + x_{jw})} \right. \\
& - \frac{1 + x_{H_k^- w}}{(x_{H_k^- w} - x_{jw})(-1 + x_{iw})} + \frac{1 + x_{H_k^- w}}{(-1 + x_{iw})(-1 + x_{jw})} \Big) \left( L_{i_2}(x_{H_k^- w}) + \ln x_{H_k^- w} \ln(1 - x_{H_k^- w}) \right) \\
& - \frac{x_{H_k^- w}^4 - x_{iw}^3 + x_{H_k^- w}^2 (x_{jw} - 1) + x_{H_k^- w} (x_{iw} - x_{H_k^- w}^2) (x_{iw} + x_{jw} - 2) \ln^2 \left( \frac{x_{H_k^- w}}{x_{iw}} \right)}{6(x_{H_k^- w} - x_{iw})(-1 + x_{iw})(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} \\
& - \frac{x_{H_k^- w} (x_{H_k^- w}^3 + 2x_{H_k^- w}^2 - 2x_{H_k^- w} - 2 + x_{iw} (1 - x_{H_k^- w} - x_{H_k^- w}^2)) \ln^2 x_{H_k^- w}}{6(x_{H_k^- w} - x_{iw})(-1 + x_{iw})(x_{H_k^- w} - x_{jw})(-1 + x_{jw})} \\
& - \frac{x_{H_k^- w} x_{jw} (1 - x_{H_k^- w}^2 + 4x_{H_k^- w} x_{iw} - x_{H_k^- w}) \ln^2 x_{H_k^- w}}{4(x_{H_k^- w} - x_{iw})(-1 + x_{iw})(x_{H_k^- w} - x_{jw})(-1 + x_{jw})} \\
& + \frac{(x_{H_k^- w}^4 - x_{iw} x_{jw}^2 + (x_{iw} + x_{jw} - 2)(x_{H_k^- w} x_{jw} - x_{H_k^- w}^3) + x_{H_k^- w}^2 (4x_{jw} - 1)(x_{iw} - 1)) \ln^2 \left( \frac{x_{H_k^- w}}{x_{jw}} \right)}{6(x_{H_k^- w} - x_{iw})(x_{iw} - x_{jw})(x_{H_k^- w} - x_{jw})(-1 + x_{jw})} \\
& + \frac{x_{iw} \ln^2 x_{iw} (x_{iw} (1 + x_{H_k^- w})^2 - 3 - x_{H_k^- w} (2 + x_{iw}^2 + x_{iw} x_{jw} + x_{jw} (1 - x_{iw}) + x_{iw}^2 (4x_{jw} - 1)))}{3(x_{H_k^- w} - x_{iw})(x_{iw} - x_{jw})(-1 + x_{iw})(-1 + x_{jw})} \\
& + \frac{x_{iw}^2 (x_{iw} + 17x_{jw} + 1 - 94x_{iw} x_{jw} - 3x_{H_k^- w}^2 (16 + 5x_{iw}) + x_{H_k^- w} (17x_{iw} + 17x_{jw} + 62))}{6(x_{H_k^- w} - x_{iw})(-1 + x_{iw})(x_{iw} - x_{jw})^2} \\
& - \frac{4(3x_{H_k^- w}^2 - 15x_{H_k^- w} x_{iw} - 2x_{iw}^2) x_{iw} \ln x_{iw}}{3(x_{H_k^- w} - x_{iw})(-1 + x_{iw})(x_{iw} - x_{jw})^2} \\
& + \frac{x_{iw}^2 ((28x_{H_k^- w} - 26x_{iw} - x_{jw}^2 + 3x_{jw} - 6) - 8x_{H_k^- w} x_{iw}) \ln^2 x_{iw}}{3(x_{H_k^- w} - x_{iw})(-1 + x_{iw})(x_{iw} - x_{jw})^2} \\
& + \frac{4x_{iw} (1 - x_{H_k^- w}) \ln^2 \left( \frac{x_{H_k^- w}}{x_{iw}} \right)}{(x_{H_k^- w} - x_{iw})(-1 + x_{iw})(x_{iw} - x_{jw})} + 2 \left( \frac{(4x_{H_k^- w}^2 - 11x_{H_k^- w} x_{iw} + x_{iw} (8x_{iw} + 8x_{jw} - x_{H_k^- w})) \ln^2 x_{iw}}{3(x_{H_k^- w} - x_{iw})(-1 + x_{iw})(x_{iw} - x_{jw})} \right. \\
& + \frac{2(7x_{H_k^- w}^2 - 44x_{H_k^- w} x_{iw} + 7x_{iw}^2) \ln x_{iw} - 8x_{iw} (x_{H_k^- w} + x_{jw}) \ln x_{iw} \ln x_{jw}}{3(x_{H_k^- w} - x_{iw})(-1 + x_{iw})(x_{iw} - x_{jw})} \\
& + \frac{2(9x_{H_k^- w}^2 - 63x_{H_k^- w} x_{iw} + 4x_{iw}^2 + 4x_{iw} (x_{H_k^- w} + x_{jw}) \ln^2 x_{jw})}{3(x_{H_k^- w} - x_{iw})(-1 + x_{iw})(x_{iw} - x_{jw})} \Big) \\
& + \frac{x_{iw} ((1 + x_{H_k^- w}) (x_{iw}^2 + 2x_{jw} + x_{iw} x_{jw}) - x_{iw} (1 + x_{H_k^- w})^2 - 4x_{iw}^2 x_{jw}) \ln^2 \left( \frac{x_{iw}}{x_{jw}} \right)}{2(x_{H_k^- w} - x_{iw})(x_{iw} - x_{jw})^2 (-1 + x_{jw})}
\end{aligned}$$

$$\begin{aligned}
& - \frac{\left( (1 + x_{H_k^- w})(x_{iw}^2(x_{iw} + x_{jw} - 1 - x_{H_k^- w}) - 2x_{iw}x_{jw}) - 4x_{iw}^3x_{jw} \right) \ln^2(x_{iw}x_{jw})}{12(x_{H_k^- w} - x_{iw})(x_{iw} - x_{jw})^2(-1 + x_{jw})} \\
& + \frac{x_{H_k^- w} \left( (1 + 2x_{H_k^- w} - x_{iw} + 2x_{jw}) \ln^2\left(\frac{x_{H_k^- w}}{x_{jw}}\right) - (1 + 2x_{H_k^- w} + x_{iw}) \ln^2\left(\frac{x_{H_k^- w}}{x_{iw}}\right) \right)}{4(x_{H_k^- w} - x_{iw})(x_{iw} - x_{jw})(-1 + x_{jw})} \\
& + \frac{2x_{H_k^- w}(\ln^2 x_{H_k^- w} + 4x_{iw} \ln^2 x_{iw} - 8x_{iw} \ln x_{iw} \ln x_{jw} - 4x_{jw} \ln^2 x_{jw} + 4x_{H_k^- w} x_{iw} \ln^2 x_{jw})}{3(x_{H_k^- w} - x_{iw})(x_{iw} - x_{jw})} \\
& + \frac{2(x_{iw} + x_{jw}) \ln^2 x_{iw} + 4(x_{H_k^- w} - 1)x_{jw} \ln x_{H_k^- w} \ln x_{jw} + 16x_{iw}x_{jw}(1 + x_{iw}) \ln x_{iw} \ln x_{jw}}{3(x_{H_k^- w} - x_{iw})(x_{iw} - x_{jw})(-1 + x_{jw})} \\
& + \frac{8(4x_{H_k^- w} - x_{jw})x_{jw} \ln x_{jw} + x_{jw} \ln^2 x_{jw}(22x_{H_k^- w} - 8x_{iw}^2 + 16x_{iw} - 6) - 6x_{iw} \ln x_{iw} \ln x_{H_k^- w}}{3(x_{H_k^- w} - x_{iw})(x_{iw} - x_{jw})(-1 + x_{jw})} \\
& + \frac{8(x_{iw}^2 \ln^2 x_{iw} + 2(x_{H_k^- w} + x_{jw}) \ln^2 x_{H_k^- w}) - 6 \ln^2 x_{H_k^- w}}{3(x_{H_k^- w} - x_{iw})(x_{iw} - x_{jw})(-1 + x_{jw})} + \frac{2 \ln^2 x_{H_k^- w}}{3(x_{iw} - 1)(x_{jw} - 1)} \\
& + \frac{(x_{iw} - x_{H_k^- w} - 4)(x_{iw} \ln^2 x_{iw} - x_{jw} \ln^2 x_{jw})}{6(-1 + x_{iw})(x_{iw} - x_{jw})(-1 + x_{jw})} + \frac{(1 + x_{H_k^- w}) \ln^2 x_{jw}}{(x_{iw} - 1)(x_{jw} - 1)(x_{jw} - x_{H_k^- w})} \\
& - \frac{(1 + x_{H_k^- w})x_{iw} \ln^2\left(\frac{x_{iw}}{x_{jw}}\right)}{(x_{iw} - 1)(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} + \frac{x_{H_k^- w}(\ln^2 x_{jw} + \ln^2 x_{iw} + \ln x_{H_k^- w} \ln\left(\frac{x_{iw}}{x_{jw}}\right))}{12(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} \\
& - \frac{x_{H_k^- w}(x_{H_k^- w} \ln x_{H_k^- w} \ln\left(\frac{x_{iw}}{x_{jw}}\right) + (x_{H_k^- w} \ln^2 x_{iw} + x_{iw}(\ln^2 x_{iw} - \ln x_{iw} \ln x_{H_k^- w}) + (x_{iw} \rightarrow x_{jw})))}{6(x_{iw} - 1)(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} \\
& + \frac{\ln^2 x_{H_k^- w}}{6(x_{H_k^- w} - x_{jw})(x_{iw} - 1)} + \frac{2(1 + x_{H_k^- w})x_{iw} \ln^2\left(\frac{x_{iw}}{x_{H_k^- w}}\right)}{3(x_{H_k^- w} - x_{iw})(-1 + x_{iw})(x_{H_k^- w} - x_{jw})} \\
& - \frac{(1 + x_{H_k^- w}) \ln^2 x_{H_k^- w}}{(x_{H_k^- w} - 1)(x_{iw} - 1)(x_{H_k^- w} - x_{jw})} + \frac{(x_{H_k^- w} - x_{iw}) \ln^2 x_{H_k^- w}}{3(x_{H_k^- w} - 1)(x_{iw} - 1)(x_{H_k^- w} - x_{jw})^2} \\
& - \frac{x_{H_k^- w}^2 \ln^2 x_{H_k^- w}}{3(x_{H_k^- w} - x_{jw})^2(x_{iw} - 1)} - \frac{x_{iw} + 140x_{H_k^- w}^2 - x_{H_k^- w}x_{iw} - 16 - 128x_{jw}}{12(x_{H_k^- w} - 1)(x_{iw} - 1)(x_{H_k^- w} - x_{jw})^2} \\
& + \frac{4x_{iw}(6(x_{H_k^- w} - x_{iw}) + 3(x_{H_k^- w} - 4x_{iw}) \ln x_{iw} + 2(x_{H_k^- w} - 3x_{iw}) \ln^2 x_{iw})}{3(x_{iw} - 1)^2(x_{iw} - x_{jw})} \\
& + \frac{4x_{iw}(6(x_{H_k^- w} - x_{iw}) + 3(x_{H_k^- w} - 4x_{iw}) \ln x_{iw} + 2(x_{H_k^- w} - 3x_{iw}) \ln^2 x_{iw})}{3(x_{H_k^- w} - x_{iw})(x_{iw} - 1)(x_{iw} - x_{jw})} \\
& - \frac{8(12x_{H_k^- w}x_{iw} + 7x_{iw}^2 \ln x_{iw})}{3(x_{H_k^- w} - x_{iw})(x_{iw} - 1)^2(x_{iw} - x_{jw})} - \frac{8(12x_{H_k^- w}x_{iw} + 7x_{iw}^2 \ln x_{iw})}{3(x_{H_k^- w} - x_{iw})^2(x_{iw} - 1)(x_{iw} - x_{jw})}
\end{aligned}$$

$$\begin{aligned}
& - \frac{8x_{H_k^- w}(\ln^2(\frac{x_{H_k^- w}}{x_{iw}}) + 2(x_{H_k^- w} + x_{jw})\ln^2(\frac{x_{H_k^- w}}{x_{jw}}) - 2(x_{iw} + x_{H_k^- w})\ln^2 x_{H_k^- w})}{3(x_{H_k^- w} - 1)(x_{H_k^- w} - x_{iw})(x_{H_k^- w} - x_{jw})} \\
& + \frac{8x_{H_k^- w}(2x_{iw}\ln x_{iw}\ln x_{H_k^- w} + 3x_{H_k^- w}\ln x_{H_k^- w})}{3(x_{H_k^- w} - 1)(x_{H_k^- w} - x_{iw})(x_{H_k^- w} - x_{jw})} + \frac{8x_{H_k^- w}x_{iw}\ln^2(\frac{x_{H_k^- w}}{x_{iw}})}{3(x_{H_k^- w} - 1)(x_{H_k^- w} - x_{jw})} \\
& + \frac{4x_{iw}(4x_{H_k^- w}^3 + x_{H_k^- w}^2(2 - 4x_{iw}) - 5x_{H_k^- w}x_{iw} + x_{iw}^2)\ln^2(\frac{x_{H_k^- w}}{x_{iw}}) - 8x_{H_k^- w}x_{iw}(x_{H_k^- w} + x_{iw})\ln^2 x_{iw}}{3(x_{H_k^- w} - 1)(x_{H_k^- w} - x_{iw})^2(x_{H_k^- w} - x_{jw})} \\
& + \frac{2(6x_{H_k^- w} - 3x_{H_k^- w}x_{iw} - 2x_{iw}^2 + 2x_{iw}^2x_{H_k^- w} - 2x_{H_k^- w}^2)\ln^2 x_{H_k^- w}}{3(x_{H_k^- w} - 1)(x_{H_k^- w} - x_{iw})^2(x_{H_k^- w} - x_{jw})} \\
& - \frac{4(x_{H_k^- w}^2 + x_{iw}(5 + x_{jw}) - 3x_{H_k^- w}(1 + 2x_{iw}))\ln^2 x_{iw}}{3(x_{H_k^- w} - 1)(x_{iw} - 1)^2(x_{jw} - 1)} + \frac{(1 + x_{H_k^- w})x_{iw}\ln^2(\frac{x_{iw}}{x_{jw}})}{3(x_{iw} - 1)(x_{iw} - x_{jw})(x_{H_k^- w} - x_{jw})} \\
& + \left(3x_{jw}^2((x_{iw} + x_{jw} - 2)(x_{H_k^- w} + 1) + 1 - x_{H_k^- w}^2 - 4x_{iw}x_{jw}) + 8x_{iw}(x_{H_k^- w}^2 - x_{H_k^- w}(6x_{iw} + x_{jw} - 4x_{jw}^2)) \right. \\
& + x_{iw}(x_{iw} - x_{jw}(1 + 4x_{jw}))\ln^2 x_{iw} + 8(2x_{iw}^2 + 15x_{H_k^- w}x_{iw} - 3x_{H_k^- w}^2)x_{jw}\ln x_{jw} - 16(x_{H_k^- w}^2 + x_{iw}(x_{iw} \\
& + x_{jw} - 4x_{jw}^2) + x_{H_k^- w}(4x_{jw}^2 + x_{jw} - 6x_{iw}))\ln x_{iw}\ln x_{jw} + 2(4x_{iw}^3 + 4x_{H_k^- w}^2(x_{iw} - 2x_{jw}) + (x_{iw} \\
& - 2 - x_{jw})x_{jw}^2 - 4x_{iw}^2x_{jw}(3 + 4x_{jw}) - 2x_{H_k^- w}(12x_{iw}^2 + x_{jw}^2 - 2x_{iw}x_{jw}(11 + 4x_{jw})))\ln^2 x_{jw}) \\
& \left. / (3(x_{H_k^- w} - x_{iw})(-1 + x_{iw})(x_{iw} - x_{jw})^2) \right) \\
& + \left(6x_{H_k^- w}x_{iw} + 12x_{H_k^- w}^2x_{iw} + 6x_{H_k^- w}^3x_{iw} - 6x_{H_k^- w}x_{iw}^2 - 6x_{H_k^- w}^2x_{iw}^2 - 12x_{H_k^- w}x_{iw}^3 + 6x_{H_k^- w}x_{jw} \right. \\
& + 12x_{H_k^- w}^2x_{jw} + 6x_{H_k^- w}^3x_{jw} - 12x_{iw}x_{jw} - 48x_{H_k^- w}x_{iw}x_{jw} - 48x_{H_k^- w}^2x_{iw}x_{jw} - 12x_{H_k^- w}^3x_{iw}x_{jw} \\
& + 48x_{H_k^- w}x_{iw}^2x_{jw} + 18x_{H_k^- w}^2x_{iw}^2x_{jw} + 6x_{iw}^3x_{jw} + 6x_{H_k^- w}x_{iw}^3x_{jw} - 6x_{H_k^- w}x_{jw}^2 - 6x_{H_k^- w}^2x_{jw}^2 \\
& + 18x_{iw}x_{jw}^2 + 18x_{iw}^2x_{jw} + 48x_{H_k^- w}x_{iw}x_{jw}^2 + 18x_{H_k^- w}^2x_{iw}x_{jw}^2 - 36x_{iw}^2x_{jw}^2 - 36x_{H_k^- w}x_{iw}^2x_{jw}^2 \\
& - 12x_{H_k^- w}x_{jw}^3 + 6x_{iw}x_{jw}^3 + 6x_{H_k^- w}x_{iw}x_{jw}^3 - (4x_{H_k^- w}^2 + 12x_{H_k^- w}(-1 + x_{iw}) \\
& - x_{iw}(3 + x_{iw}))(x_{iw} - x_{jw})^2(-1 + x_{jw})\ln^2 x_{H_k^- w} - (x_{H_k^- w}^3(-3x_{iw} + x_{iw}^2 + x_{jw} + 2x_{iw}x_{jw} - x_{jw}^2) \\
& - x_{H_k^- w}^2(2x_{iw}^3 + x_{iw}^2(1 - 6x_{jw}) + x_{jw}(2 + x_{jw} - 3x_{jw}^2) + x_{iw}(2 - 16x_{jw} + 17x_{jw}^2)) \\
& + x_{iw}(x_{iw}^3(4 - 5x_{jw}) + x_{iw}x_{jw}(-9 + 14x_{jw} - 9x_{jw}^2) + x_{iw}^2(-4 + 5x_{jw} + 4x_{jw}^2) + x_{jw}(2 + 3x_{jw} \\
& - 7x_{jw}^2 + 2x_{jw}^3)) + x_{H_k^- w}(x_{iw}^4 + 3x_{iw}^3(-2 + x_{jw}) + x_{iw}^2(13 - 22x_{jw} + 7x_{jw}^2) - x_{jw}(1 + 3x_{jw}
\end{aligned}$$

$$\begin{aligned}
& -6x_{jw}^2 + 2x_{jw}^3) + x_{iw}(-1 + 4x_{jw} - 6x_{jw}^2 + 7x_{jw}^3))) \ln^2 x_{iw} + 8(x_{H_k^- w} - x_{iw} - x_{H_k^- w} x_{iw} \\
& + x_{iw}^2) x_{iw}^2 \ln x_{jw} + (-2x_{H_k^- w} x_{iw} x_{jw} + 6x_{H_k^- w}^2 x_{iw} x_{jw} + 2x_{iw}^2 x_{jw} - 12x_{H_k^- w} x_{iw}^2 x_{jw} \\
& - 6x_{H_k^- w}^2 x_{iw}^2 x_{jw} + 6x_{iw}^3 x_{jw} + 14x_{H_k^- w} x_{iw}^3 x_{jw} - 8x_{iw}^4 x_{jw} - 6x_{H_k^- w} x_{jw}^2 - 6x_{H_k^- w}^2 x_{jw}^2 \\
& + 6x_{iw} x_{jw}^2 + 8x_{H_k^- w} x_{iw} x_{jw}^2 + 6x_{H_k^- w}^2 x_{iw} x_{jw}^2 - 2x_{iw}^2 x_{jw}^2 - 2x_{H_k^- w} x_{iw}^2 x_{jw}^2 - 4x_{iw}^3 x_{jw}^2 + 12x_{H_k^- w} x_{jw}^3 \\
& - 12x_{iw} x_{jw}^3 - 12x_{H_k^- w} x_{iw} x_{jw}^3 + 12x_{iw}^2 x_{jw}^3) \ln x_{jw} + (x_{H_k^- w} x_{iw} + 2x_{H_k^- w}^2 x_{iw} - x_{H_k^- w}^3 x_{iw} \\
& - x_{H_k^- w} x_{iw}^2 - x_{H_k^- w}^2 x_{iw}^2 + x_{H_k^- w}^3 x_{iw}^2 - x_{iw}^3 - x_{H_k^- w}^2 x_{iw}^3 + x_{iw}^4 + x_{H_k^- w} x_{jw} + 2x_{H_k^- w}^2 x_{jw} \\
& + 3x_{H_k^- w}^3 x_{jw} - 2x_{iw} x_{jw} - 4x_{H_k^- w} x_{iw} x_{jw} - 4x_{H_k^- w}^2 x_{iw} x_{jw} - 2x_{H_k^- w}^3 x_{iw} x_{jw} + 3x_{iw}^2 x_{jw} \\
& + 4x_{H_k^- w}^2 x_{iw}^2 x_{jw} - x_{iw}^4 x_{jw} - 9x_{H_k^- w} x_{jw}^2 - 9x_{H_k^- w}^2 x_{jw}^2 - x_{H_k^- w}^3 x_{jw}^2 + 8x_{iw} x_{jw}^2 + 16x_{H_k^- w} x_{iw} x_{jw}^2 \\
& + 6x_{H_k^- w}^2 x_{iw} x_{jw}^2 - 7x_{iw}^2 x_{jw}^2 - 6x_{H_k^- w} x_{iw}^2 x_{jw}^2 + 2x_{iw}^3 x_{jw}^2 + 12x_{H_k^- w} x_{jw}^3 + x_{H_k^- w}^2 x_{jw}^3 - 10x_{iw} x_{jw}^3 \\
& - 10x_{H_k^- w} x_{iw} x_{jw}^3 + 7x_{iw}^2 x_{jw}^3) \ln^2 x_{jw} + 2 \ln x_{iw} ((7 + 3x_{H_k^- w} - 10x_{iw}) x_{iw} (x_{H_k^- w} - x_{jw}) (x_{iw} - x_{jw}) \\
& (-1 + x_{jw}) + (x_{H_k^- w}^3 (-x_{iw} + x_{iw}^2 + (-1 + x_{jw}) x_{jw}) + x_{iw} (-x_{iw}^2 (1 - 2x_{jw})^2) + x_{iw}^3 (-1 + x_{jw}) \\
& + 2(-1 + x_{jw})^2 x_{jw} (1 + x_{jw}) + x_{iw} x_{jw} (-7 + 15x_{jw} - 7x_{jw}^2)) - x_{H_k^- w}^2 (x_{iw}^2 + x_{iw}^3 + x_{jw} (2 - 3x_{jw} \\
& + x_{jw}^2) + 2x_{iw} (1 - 6x_{jw} + 4x_{jw}^2)) - x_{H_k^- w} (2x_{iw}^3 (-2 + x_{jw}) + x_{iw}^2 (-3 + 18x_{jw} - 14x_{jw}^2) \\
& + x_{iw} (1 - 12x_{jw} + 18x_{jw}^2 - 6x_{jw}^3) + x_{jw} (1 + x_{jw} - 4x_{jw}^2 + 2x_{jw}^3))) \ln x_{jw}) \\
& - 2(x_{iw} - x_{jw}) \ln x_{H_k^- w} ((-1 + x_{jw}) (-2x_{H_k^- w}^3 + 4x_{H_k^- w}^2 (x_{iw} + x_{jw}) - 2x_{H_k^- w} x_{iw} (-4 + 6x_{iw} + 3x_{jw}) \\
& + x_{iw} (3x_{iw}^2 - 7x_{jw} + x_{iw} (-1 + 9x_{jw}))) \ln x_{iw} + (-1 + x_{iw}) (-((3x_{H_k^- w} + 4x_{iw}) (x_{iw} - x_{jw}) (-1 + x_{jw})) \\
& + 2(x_{H_k^- w} - x_{iw}) (x_{H_k^- w}^2 - 3x_{H_k^- w} x_{jw} + 2x_{jw} (-2 + 3x_{jw})) \ln x_{jw})) / (4(-1 + x_{iw}) \\
& (-x_{H_k^- w} + x_{iw}) (x_{H_k^- w} - x_{jw}) (x_{iw} - x_{jw})^2 (-1 + x_{jw})) + (x_{iw} \leftrightarrow x_{jw}) \Big), \tag{74}
\end{aligned}$$

$$WH_3 = -2WH_2, \tag{75}$$

$$\begin{aligned}
HH_1 = & -\left(\frac{2}{3\sin^4\beta} x_{iw}^2 x_{jw}^2 (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 F_A^0 - \frac{1}{6\sin^4\beta} x_{iw} x_{jw} (x_{iw} + x_{jw}) (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 (F_A^{1a} + F_A^{1b} \right. \\
& \left. - F_A^{1c}) + \frac{1}{12\sin^3\beta} (h_d + h_b) x_{iw}^{\frac{1}{2}} x_{jw} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2l} (F_A^{2a} + F_A^{2b} + F_A^{2c} + 2F_A^{2d} - 2F_A^{2e} \right.
\end{aligned}$$

$$\begin{aligned}
& -2F_A^{2f}) \Big) (x_{iw}, x_{jw}, x_{H_l^- w}, 0, x_{iw}, x_{jw}, x_{H_k^- w}) \\
& + \frac{4}{3 \sin^4 \beta} x_{iw} x_{jw} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \Big( - \sum_{\sigma=u^i, H_k^-, H_l^-} \Big( \frac{x_\sigma \ln^2 x_\sigma}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + (x_{H_k^- w} + x_{jw}) \frac{\mathcal{R}_{i_2}(\frac{x_{jw}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \Big) \\
& - \frac{\mathcal{R}_{i_2}(\frac{x_{jw}}{x_{iw}}) - \mathcal{R}_{i_2}(\frac{x_{jw}}{x_{H_l^- w}})}{(x_{H_k^- w} - x_{H_l^- w})(x_{iw} - x_{H_l^- w})} \Big) \\
& - \frac{x_{iw} x_{jw}}{3 \sin^4 \beta} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \Big( \sum_{\sigma=u^i, u^j, H_l^-} \Big( - \frac{3x_\sigma \ln^2 x_\sigma}{2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + 2(x_{H_k^- w} \\
& + x_{H_l^- w}) \frac{\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + \frac{x_\sigma (\ln x_\sigma - \ln^2 x_\sigma - \Upsilon(\frac{x_{H_k^- w}}{x_\sigma}))}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \Big) + 2 \frac{\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_{iw}}) - \mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_{jw}})}{-x_{iw} + x_{jw}} \Big) \\
& - \frac{32}{3 \sin^2 \beta} x_{iw} x_{jw} (h_b^2 + h_d^2) \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2l} \Big( \sum_{\sigma=u^j, H_k^-, H_l^-} \frac{x_\sigma (\ln x_\sigma - 2 \ln^2 x_\sigma - \Upsilon(\frac{x_{iw}}{x_\sigma}))}{2(x_{iw} - x_\sigma)^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \\
& + \frac{(-1 + 3 \ln x_{iw} + 2 \ln^2 x_{iw})}{2(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} + \frac{x_{iw}(-\ln x_{iw} + 2 \ln^2 x_{iw})}{2(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})^2} \\
& + \frac{x_{iw}(-\ln x_{iw} + 2 \ln^2 x_{iw})}{2(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})^2(x_{jw} - x_{iw})} + \frac{x_{iw}(-\ln x_{iw} + 2 \ln^2 x_{iw})}{2(x_{H_k^- w} - x_{iw})^2(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} \Big) \\
& - \frac{32}{3 \sin^4 \beta} x_{iw}^2 x_{jw} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \Big( \sum_{\sigma=u^j, H_k^-, H_l^-} \frac{x_{H_k^- w} x_\sigma (-\ln x_\sigma - \ln^2 x_\sigma - \Upsilon(\frac{x_{iw}}{x_\sigma}))}{2(x_{iw} - x_\sigma)^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \\
& + \frac{x_{H_k^- w} (1 - 3 \ln x_{iw} + \ln^2 x_{iw})}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} + \frac{x_{H_k^- w} x_{iw} (\ln x_{iw} + \ln^2 x_{iw})}{2(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})^2} \\
& + \frac{x_{H_k^- w} x_{iw} (\ln x_{iw} + \ln^2 x_{iw})}{2(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})^2(x_{jw} - x_{iw})} + \frac{x_{H_k^- w} x_{iw} (\ln x_{iw} + \ln^2 x_{iw})}{2(x_{H_k^- w} - x_{iw})^2(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \Big( \frac{(x_{H_l^- w} (\ln x_{H_l^- w} + \ln^2 x_{H_l^- w} + \Upsilon(\frac{x_{iw}}{x_{H_l^- w}})))}{(-x_{H_l^- w} + x_{iw})^2 (-x_{H_l^- w} + x_{jw})} + (x_{H_l^- w} \rightarrow x_{jw}) \Big) \\
& - \Big( \frac{1 + 3 \ln x_{iw} + \ln^2 x_{iw}}{(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} + \frac{x_{iw} (\ln x_{iw} + \ln^2 x_{iw})}{(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})^2} \\
& + \frac{x_{iw} (\ln x_{iw} + \ln^2 x_{iw})}{(x_{H_l^- w} - x_{iw})^2(x_{jw} - x_{iw})} \Big) \Big) + \frac{16}{3 \sin^2 \beta} (h_b^2 + h_d^2) x_{iw} x_{jw} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2l} \Big(
\end{aligned}$$

$$\begin{aligned}
& \sum_{\sigma=u^j, H_k^-, H_l^-} \frac{x_\sigma(2x_{iw} \ln x_\sigma - 2(x_{H_k^-w} + x_{iw}) \ln^2 x_\sigma - x_{iw} \ln^2(x_{iw}x_\sigma) - 2(x_{H_k^-w} - x_{iw})\Upsilon(\frac{x_{iw}}{x_\sigma}))}{2(x_{iw} - x_\sigma)^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \\
& + \frac{(-x_{iw} + (x_{H_k^-w} + 4x_{iw}) \ln x_{iw} + (x_{H_k^-w} + 5x_{iw}) \ln^2 x_{iw})}{(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \frac{-x_{iw}^2 \ln x_{iw} + (3x_{iw} + x_{H_k^-w})x_{iw} \ln^2 x_{iw}}{(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})^2} + \frac{-x_{iw}^2 \ln x_{iw} + (3x_{iw} + x_{H_k^-w})x_{iw} \ln^2 x_{iw}}{(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})^2(x_{jw} - x_{iw})} \\
& + \frac{-x_{iw}^2 \ln x_{iw} + (3x_{iw} + x_{H_k^-w})x_{iw} \ln^2 x_{iw}}{(x_{H_k^-w} - x_{iw})^2(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})} + \left( \frac{x_{H_l^-w}(\ln^2 x_{H_l^-w} + \Upsilon(\frac{x_{iw}}{x_{H_l^-w}}))}{(-x_{H_l^-w} + x_{iw})^2(-x_{H_l^-w} + x_{jw})} + (x_{H_l^-w} \leftrightarrow x_{jw}) \right) \\
& - \frac{2 \ln x_{iw} + \ln^2 x_{iw}}{(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})} - \frac{x_{iw} \ln^2 x_{iw}}{(x_{H_l^-w} - x_{iw})^2(x_{jw} - x_{iw})} \Big) + \frac{4}{3 \sin^4 \beta} x_{iw}^2 x_{jw} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \Big( \\
& \sum_{\sigma=u^j, H_k^-, H_l^-} \frac{x_\sigma(-(3x_{H_k^-w} - 2x_{iw}) \ln x_\sigma - 2(x_{H_k^-w} + x_{iw})(\ln^2 x_\sigma + \Upsilon(\frac{x_{iw}}{x_{H_k^-w}})))}{2(x_{iw} - x_\sigma)^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \\
& + \frac{(3x_{H_k^-w} - 2x_{iw} + (7x_{H_k^-w} - 11x_{iw}) \ln x_{iw} + 2(x_{H_k^-w} + 5x_{iw}) \ln^2 x_{iw})}{2(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \frac{x_{iw}(3x_{H_k^-w} - 2x_{iw}) \ln x_\sigma + 2x_{iw}(x_{H_k^-w} + 3x_{iw}) \ln^2 x_{iw}}{2(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})^2} \\
& + \frac{x_{iw}(3x_{H_k^-w} - 2x_{iw}) \ln x_\sigma + 2x_{iw}(x_{H_k^-w} + 3x_{iw}) \ln^2 x_{iw}}{2(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})^2(x_{jw} - x_{iw})} \\
& + \frac{x_{iw}(3x_{H_k^-w} - 2x_{iw}) \ln x_\sigma + 2x_{iw}(x_{H_k^-w} + 3x_{iw}) \ln^2 x_{iw}}{2(x_{H_k^-w} - x_{iw})^2(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \left( \frac{-\frac{3}{2}x_{iw} \ln x_{iw} + x_{iw} \ln^2 x_{iw} + x_{H_l^-w} \Upsilon(\frac{x_{iw}}{x_{H_l^-w}})}{(-x_{H_l^-w} + x_{iw})^2(-x_{H_l^-w} + x_{jw})} + (x_{H_l^-w} \leftrightarrow x_{jw}) \right) \\
& + \frac{-3 - 7 \ln x_{iw} - 2 \ln^2 x_{iw}}{2(-x_{H_l^-w} + x_{iw})(-x_{iw} + x_{jw})} + \frac{3x_{H_l^-w} \ln x_{H_l^-w} + 2x_{H_l^-w} \ln^2 x_{H_l^-w}}{2(-x_{H_l^-w} + x_{iw})^2(-x_{iw} + x_{jw})} \Big) \\
& + \frac{4}{3 \sin^2 \beta} x_{iw} x_{jw} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \Big( \sum_{\sigma=u^j, H_k^-, H_l^-} \left( -\frac{x_{H_k^-w}^2 x_{iw} x_\sigma \ln^2(x_\sigma x_{iw})}{2(x_{iw} - x_\sigma)^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \\
& \left. + \frac{x_\sigma((-3x_{H_k^-w}^2 + 5x_{H_k^-w} x_{iw} - 3x_{iw}^2) \ln x_\sigma - 2(x_{H_k^-w}^2 + x_{iw}^2)(\ln^2 x_\sigma + \Upsilon(\frac{x_{iw}}{x_\sigma})))}{2(x_{iw} - x_\sigma)^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right) \Big)
\end{aligned}$$



$$\begin{aligned}
& + \frac{3x_{H_k^-w}^2 - 5x_{H_k^-w}x_{iw} + 3x_{iw}^2 + (7x_{H_k^-w}^2 - 2x_{H_k^-w}x_{iw} + 7x_{iw}^2) \ln x_{iw}}{2(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \frac{(x_{H_k^-w}^2 + 4x_{H_k^-w}x_{iw} + x_{iw}^2) \ln^2 x_{iw}}{(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \frac{(3x_{H_k^-w}^2x_{iw} - 5x_{H_k^-w}x_{iw}^2 + 3x_{iw}^3) \ln x_{iw} + 2(x_{H_k^-w}^2x_{iw} + x_{iw}^3 + x_{H_k^-w}x_{iw}^2) \ln^2 x_{iw}}{(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})^2} \\
& + \frac{(3x_{H_k^-w}^2x_{iw} - 5x_{H_k^-w}x_{iw}^2 + 3x_{iw}^3) \ln x_{iw} + 2(x_{H_k^-w}^2x_{iw} + x_{iw}^3 + x_{H_k^-w}x_{iw}^2) \ln^2 x_{iw}}{(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})^2(x_{jw} - x_{iw})} \\
& + \frac{(3x_{H_k^-w}^2x_{iw} - 5x_{H_k^-w}x_{iw}^2 + 3x_{iw}^3) \ln x_{iw} + 2(x_{H_k^-w}^2x_{iw} + x_{iw}^3 + x_{H_k^-w}x_{iw}^2) \ln^2 x_{iw}}{(x_{H_k^-w} - x_{iw})^2(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \left( \frac{(3x_{H_k^-w} - 5x_{iw})x_{H_l^-w} \ln x_{H_l^-w} + 2(x_{H_k^-w} + x_{H_l^-w})x_{H_l^-w} \ln^2 x_{H_l^-w} - x_{H_l^-w}x_{iw} \ln^2(x_{H_l^-w}x_{iw})}{2(-x_{H_l^-w} + x_{iw})^2(-x_{H_l^-w} + x_{jw})} \right. \\
& + \frac{x_{H_l^-w}(x_{H_k^-w} + x_{H_l^-w})\Upsilon\left(\frac{x_{iw}}{x_{H_l^-w}}\right)}{(-x_{H_l^-w} + x_{iw})^2(-x_{H_l^-w} + x_{jw})} + (x_{H_l^-w} \leftrightarrow x_{jw}) \Big) \\
& + \frac{-(x_{H_k^-w} + x_{H_l^-w})(3 + 7 \ln x_{iw} + \ln^2 x_{iw}) + x_{iw}(5 + 9 \ln x_{iw} + 4 \ln^2 x_{iw})}{2(-x_{H_l^-w} + x_{iw})(-x_{iw} + x_{jw})} \\
& + \frac{(x_{H_k^-w} + x_{H_l^-w})x_{iw}(-3 \ln x_{iw} - 2 \ln^2 x_{iw}) + x_{iw}^2(5 \ln x_{iw} + 4 \ln^2 x_{iw})}{2(-x_{H_l^-w} + x_{iw})(-x_{iw} + x_{jw})^2} \\
& + \frac{(x_{H_k^-w} + x_{H_l^-w})x_{iw}(-3 \ln x_{iw} - 2 \ln^2 x_{iw}) + x_{iw}^2(5 \ln x_{iw} + 4 \ln^2 x_{iw})}{2(-x_{H_l^-w} + x_{iw})^2(-x_{iw} + x_{jw})} \\
& + \frac{3 + 7 \ln x_{iw} + \ln^2 x_{iw}}{2(x_{jw} - x_{iw})} \\
& + \frac{\left( x_{iw}(3 \ln x_{iw} + 2 \ln^2 x_{iw}) + (x_{iw} \leftrightarrow x_{jw}) \right) - 2x_{jw}\Upsilon\left(\frac{x_{iw}}{x_{jw}}\right)}{2(x_{jw} - x_{iw})^2} \\
& + \frac{3x_{H_l^-w}^2 \ln x_{H_l^-w}}{2(-x_{H_l^-w} + x_{iw})^2(-x_{H_l^-w} + x_{jw})} + \frac{3x_{H_l^-w}x_{jw} \ln x_{jw}}{2(-x_{jw} + x_{iw})^2(x_{H_l^-w} - x_{jw})} \Big) \\
& + \frac{8}{3 \sin^4 \beta} x_{iw}^2 x_{jw} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \left( - \sum_{\sigma=u^i, w^j, H_l^-} \frac{x_\sigma \mathcal{R}_{i_2}\left(\frac{x_{iw}}{x_\sigma}\right)}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \\
& \left. \sum_{\sigma=u^i, w^j, H_k^-, H_l^-} \frac{x_\sigma(-2 \ln x_\sigma + \ln^2 x_\sigma + 2(x_{H_k^-w} - x_{iw})\mathcal{R}_{i_2}\left(\frac{x_{iw}}{x_\sigma}\right) + 2\Upsilon\left(\frac{x_{iw}}{x_\sigma}\right))}{2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{x_{H_l^- w} (5 \ln x_{H_l^- w} + \ln^2 x_{H_l^- w})}{2(x_{H_k^- w} - x_{H_l^- w})(x_{iw} - x_{H_l^- w})(x_{jw} - x_{H_l^- w})} - \frac{x_{H_l^- w} (5 \ln x_{H_l^- w} + \ln^2 x_{H_l^- w})}{2(x_{H_k^- w} - x_{H_l^- w})(x_{iw} - x_{H_l^- w})^2} \\
& - \frac{16}{3 \sin^4 \beta} x_{iw} x_{jw} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \left( \right. \\
& \quad \sum_{\sigma=u^i, u^j, H_l^-} \frac{x_\sigma (3 \ln x_\sigma + 3 \ln^2 x_\sigma - 2(x_{H_k^- w} - x_{iw} + x_\sigma) \mathcal{R}_{i_2}(\frac{x_{iw}}{x_\sigma}) + 2\Upsilon(\frac{x_{iw}}{x_\sigma}))}{2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \\
& \quad + \sum_{\sigma=u^i, u^j, H_k^-, H_l^-} \frac{x_{H_k^- w} x_\sigma (-6 \ln x_\sigma - 3 \ln^2 x_\sigma + 2(x_{H_k^- w} - x_{iw}) \mathcal{R}_{i_2}(\frac{x_{iw}}{x_\sigma}) - 2\Upsilon(\frac{x_{iw}}{x_\sigma}))}{2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \\
& \quad + \frac{x_{H_k^- w} x_{H_l^- w} (-\ln x_{H_l^- w} + \ln^2 x_{H_l^- w})}{2(x_{H_k^- w} - x_{H_l^- w})(x_{iw} - x_{H_l^- w})(x_{jw} - x_{H_l^- w})} - \frac{x_{H_k^- w} x_{H_l^- w} (4 \ln x_{H_l^- w} + \ln^2 x_{H_l^- w})}{2(x_{H_k^- w} - x_{H_l^- w})(x_{iw} - x_{H_l^- w})^2} \\
& \quad + \frac{1}{3 \sin^4 \beta} x_{iw} x_{jw}^2 (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \left( - \sum_{\sigma=u^i, u^j, H_l^-} \left( \frac{\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \right. \\
& \quad \left. \left. + \frac{(x_{H_l^- w} - x_{jw})(\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_\sigma}) - \mathcal{R}_{i_2}(\frac{x_{jw}}{x_\sigma})) + x_\sigma (\Upsilon(\frac{x_{H_k^- w}}{x_\sigma}) - \Upsilon(\frac{x_{jw}}{x_\sigma}))}{(x_{H_k^- w} - x_{jw}) \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right) \right. \\
& \quad \left. + \frac{\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_{iw}}) - \mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_{jw}}) - \mathcal{R}_{i_2}(\frac{x_{jw}}{x_{iw}})}{(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} \right) \\
& - \frac{1}{3 \sin^4 \beta} x_{iw} x_{jw} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \left( \sum_{\sigma=u^i, u^j, H_l^-} \left( - \frac{x_\sigma (x_{H_k^- w} \ln^2 (x_{H_k^- w} x_\sigma) - x_{jw} \ln (x_{jw} x_\sigma))}{(x_{H_k^- w} - x_{jw}) \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \right. \\
& \quad \left. \left. - \frac{x_\sigma \ln^2 x_\sigma - 2(x_{jw} + x_{H_k^- w}) \mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_\sigma}) - 4x_\sigma \Upsilon(\frac{x_{H_k^- w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right) \right. \\
& \quad \left. + \frac{(2x_{jw}^2 - 6x_{H_l^- w}^2)(\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_\sigma}) - \mathcal{R}_{i_2}(\frac{x_{jw}}{x_\sigma})) + 8x_{H_l^- w} x_\sigma (\Upsilon(\frac{x_{H_k^- w}}{x_\sigma}) - \Upsilon(\frac{x_{jw}}{x_\sigma}))}{(x_{H_k^- w} - x_{jw}) \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \\
& \quad \left. - \frac{6(x_{H_l^- w} + x_{jw})(\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_{iw}}) - \mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_{jw}}) - \mathcal{R}_{i_2}(\frac{x_{jw}}{x_{iw}}))}{(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} \right. \\
& \quad \left. - \frac{8(x_{iw} \Upsilon(\frac{x_{H_k^- w}}{x_{iw}}) - x_{jw} \Upsilon(\frac{x_{H_k^- w}}{x_{jw}}))}{(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} + 6 \frac{\mathbb{A}(x_{iw}, x_{H_k^- w}) - \mathbb{A}(x_{iw}, x_{jw})}{x_{H_k^- w} - x_{jw}} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{4}{3 \sin^4 \beta} x_{iw} x_{jw} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \left( \frac{\mathcal{A}(x_{iw}, x_{H_k^- w}) - \mathcal{A}(x_{iw}, x_{jw})}{x_{H_k^- w} - x_{jw}} - \frac{\mathcal{R}_{i2}(\frac{x_{jw}}{x_{H_l^- w}}) - \mathcal{R}_{i2}(\frac{x_{jw}}{x_{iw}})}{x_{H_l^- w} - x_{iw}} \right. \\
& - \frac{(x_{H_k^- w} + x_{H_l^- w})(-\mathcal{R}_{i2}(\frac{x_{H_k^- w}}{x_{H_l^- w}}) - \mathcal{R}_{i2}(\frac{x_{H_k^- w}}{x_{iw}}) - \mathcal{R}_{i2}(\frac{x_{jw}}{x_{H_l^- w}}) + \mathcal{R}_{i2}(\frac{x_{jw}}{x_{iw}}))}{(x_{H_l^- w} - x_{iw})(x_{H_k^- w} - x_{jw})} \\
& \left. + \frac{(x_{iw} - x_{H_l^- w})(-\Upsilon(\frac{x_{H_k^- w}}{x_{H_l^- w}}) + \Upsilon(\frac{x_{jw}}{x_{H_l^- w}}) + (x_{H_l^- w} \rightarrow x_{iw}))}{(x_{H_l^- w} - x_{iw})(x_{H_k^- w} - x_{jw})} \right) + (x_{iw} \leftrightarrow x_{jw}), \tag{76}
\end{aligned}$$

$$\begin{aligned}
HH_2 = & -\frac{1}{12 \sin^2 \beta} h_b h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2l} \left( (x_{iw}^2 x_{jw} + x_{iw} x_{jw}^2) F_A^0 + (x_{iw} + x_{jw})^2 (F_A^{1a} + F_A^{1b} - F_A^{1c}) \right. \\
& \left. + (x_{iw} + x_{jw}) F_A^{2d} \right) (x_{iw}, x_{jw}, x_{H_l^- w}, 0, x_{iw}, x_{jw}, x_{H_k^- w}) \\
& + \frac{1}{3 \sin^2 \beta} (x_{iw} + x_{jw}) h_b h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \left[ - \sum_{\sigma=u^i, u^j, H_l^-} \frac{x_{H_l^- w} \mathcal{R}_{i2}(\frac{x_{H_k^- w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} - \frac{\mathcal{R}_{i2}(\frac{x_{H_k^- w}}{x_{iw}}) - \mathcal{R}_{i2}(\frac{x_{H_k^- w}}{x_{jw}})}{-x_{iw} + x_{jw}} \right. \\
& + 2 \left( \frac{\mathcal{R}_{i2}(\frac{x_{jw}}{x_{iw}}) - \mathcal{R}_{i2}(\frac{x_{jw}}{x_{H_l^- w}})}{(x_{H_k^- w} - x_{H_l^- w})(x_{iw} - x_{H_l^- w})} + \sum_{\sigma=u^i, H_k^-, H_l^-} \left( \frac{x_\sigma \ln^2 x_\sigma}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + (x_{H_k^- w} + x_{jw}) \frac{\mathcal{R}_{i2}(\frac{x_{jw}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right) \right) \\
& + \sum_{\sigma=u^j, H_k^-, H_l^-} \left( - \frac{x_{H_l^- w} x_{iw} \ln^2 (x_{H_l^- w} x_\sigma)}{(x_{iw} - x_\sigma)^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \\
& \left. + \frac{x_\sigma ((x_{H_l^- w}^2 + x_{H_l^- w} x_{iw} + x_{iw}^2) \ln x_\sigma - 2(x_{H_l^- w}^2 + x_{iw}^2)(\ln^2 x_\sigma + \Upsilon(\frac{x_{iw}}{x_\sigma})))}{(x_{iw} - x_\sigma)^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right) \\
& + \frac{x_{H_k^- w} x_{iw} (-1 + 2 \ln x_{iw} + 8 \ln^2 x_{iw}) + (x_{H_k^- w}^2 + x_{iw}^2) (-1 + 3 \ln x_{iw} + 2 \ln^2 x_{iw})}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \frac{(x_{H_k^- w}^2 + x_{iw}^2) x_{iw} (-\ln x_{iw} + 2 \ln^2 x_{iw}) + 3 x_{H_k^- w} x_{iw}^2 \ln^2 x_{iw}}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})^2} \\
& + \frac{(x_{H_k^- w}^2 + x_{iw}^2) x_{iw} (-\ln x_{iw} + 2 \ln^2 x_{iw}) + 3 x_{H_k^- w} x_{iw}^2 \ln^2 x_{iw}}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})^2 (x_{jw} - x_{iw})} \\
& + \frac{(x_{H_k^- w}^2 + x_{iw}^2) x_{iw} (-\ln x_{iw} + 2 \ln^2 x_{iw}) + 3 x_{H_k^- w} x_{iw}^2 \ln^2 x_{iw}}{(x_{H_k^- w} - x_{iw})^2 (x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \frac{(x_{H_k^- w} + x_{H_l^- w})(1 - 3 \ln x_{iw} - 2 \ln^2 x_{iw}) + x_{iw}(10 \ln x_{iw} + 4 \ln^2 x_{iw})}{(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})}
\end{aligned}$$

$$\begin{aligned}
& + \left( \frac{(x_{H_k^- w} + x_{H_l^- w})x_{H_l^- w}(-\ln x_{H_l^- w} + 2\ln^2 x_{H_l^- w} + 2\Upsilon(\frac{x_{iw}}{x_{H_l^- w}}))}{(-x_{H_l^- w} + x_{iw})^2(x_{jw} - x_{H_l^- w})} \right. \\
& + \frac{x_{H_l^- w}x_{iw}(\ln x_{H_l^- w} + \ln^2(x_{H_l^- w}x_{iw}))}{(-x_{H_l^- w} + x_{iw})^2(x_{jw} - x_{H_l^- w})} + (x_{H_l^- w} \leftrightarrow x_{jw}) \\
& + \frac{x_{iw}(x_{H_k^- w} + x_{H_l^- w})(\ln x_{iw} - 2\ln^2 x_{iw}) + 3x_{iw}^2 \ln^2 x_{iw}}{(-x_{H_l^- w} + x_{iw})^2(x_{jw} - x_{H_l^- w})} \\
& + \frac{x_{iw}(x_{H_k^- w} + x_{H_l^- w})(\ln x_{iw} - 2\ln^2 x_{iw}) + 3x_{iw}^2 \ln^2 x_{iw}}{(-x_{jw} + x_{iw})^2(-x_{jw} + x_{H_l^- w})} \\
& + \frac{-1 + 3\ln x_{iw} + 2\ln^2 x_{iw}}{2(x_{jw} - x_{iw})} + \frac{-x_{iw} \ln x_{iw} + 2x_{iw} \ln^2 x_{iw} + x_{jw}(\ln x_{jw} - 2\ln^2 x_{jw})}{(x_{jw} - x_{iw})^2} \\
& + \frac{21(\mathcal{A}(x_{iw}, x_{H_k^- w}) - \mathcal{A}(x_{iw}, x_{jw}))}{16(x_{H_k^- w} - x_{jw})} \\
& - 4 \left( \sum_{\sigma=u^i, u^j, H_k^-, H_l^-} \frac{x_{H_k^- w}x_\sigma(2\ln x_\sigma - 3\ln^2 x_\sigma + 2(x_{H_k^- w} - x_{iw})\mathcal{R}_{i_2}(\frac{x_{iw}}{x_\sigma}) + \Upsilon(\frac{x_{iw}}{x_\sigma}))}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \\
& - \sum_{\sigma=u^i, u^j, H_l^-} \frac{x_\sigma(2\ln x_\sigma - 3\ln^2 x_\sigma + 2(x_{H_k^- w} - x_{iw})\mathcal{R}_{i_2}(\frac{x_{iw}}{x_\sigma}) + \Upsilon(\frac{x_{iw}}{x_\sigma}))}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \\
& - \frac{2\mathcal{R}_{i_2}(\frac{x_{iw}}{x_{jw}})}{x_{iw} - x_{jw}} + \frac{x_{H_k^- w}x_{H_l^- w}(x_{iw} - x_{jw})\ln^2 x_{H_l^- w}}{(x_{H_k^- w} - x_{H_l^- w})(x_{iw} - x_{H_l^- w})^2(x_{jw} - x_{H_l^- w})} \Big) \\
& - \frac{x_{H_l^- w}(x_{H_l^- w} - x_{jw})}{x_{H_k^- w} - x_{jw}} \sum_{\sigma=u^i, u^j, H_l^-} \frac{\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_\sigma}) - \mathcal{R}_{i_2}(\frac{x_{jw}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + \frac{x_{iw}\Upsilon(\frac{x_{H_k^- w}}{x_{iw}}) - x_{jw}\Upsilon(\frac{x_{H_k^- w}}{x_{jw}})}{(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} \\
& + \frac{x_{H_l^- w}}{x_{H_k^- w} - x_{jw}} \sum_{\sigma=u^i, u^j, H_l^-} \frac{x_\sigma(\Upsilon(\frac{x_{H_k^- w}}{x_\sigma}) - \Upsilon(\frac{x_{jw}}{x_\sigma}))}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} - \frac{x_{H_l^- w}(\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_{iw}}) - \mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_{jw}}) - \mathcal{R}_{i_2}(\frac{x_{jw}}{x_{iw}}))}{(x_{H_k^- w} - x_{jw})^2(x_{iw} - x_{jw})} \\
& + \frac{5}{16} \left( \frac{\mathcal{R}_{i_2}(\frac{x_{jw}}{x_{H_l^- w}}) - \mathcal{R}_{i_2}(\frac{x_{jw}}{x_{iw}})}{x_{iw} - x_{H_l^- w}} - \frac{(-\Upsilon(\frac{x_{H_k^- w}}{x_{H_l^- w}}) + \Upsilon(\frac{x_{jw}}{x_{H_l^- w}}) + (x_{H_l^- w} \rightarrow x_{iw}))}{(x_{H_k^- w} - x_{jw})} \right. \\
& \left. - \frac{(x_{H_k^- w} + x_{H_l^- w})(-\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_{H_l^- w}}) - \mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_{iw}}) - \mathcal{R}_{i_2}(\frac{x_{jw}}{x_{H_l^- w}}) + \mathcal{R}_{i_2}(\frac{x_{jw}}{x_{iw}}))}{(x_{H_l^- w} - x_{iw})(x_{H_k^- w} - x_{jw})} \right) \Big] \\
& - \frac{x_{iw}(x_{iw} + x_{jw})}{3\sin^2 \beta} h_b h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \Big[
\end{aligned}$$

$$\begin{aligned}
& -2 \sum_{\sigma=u^j, H_k^-, H_l^-} \frac{(3x_{H_k^-w} - x_{iw})x_\sigma(\ln x_\sigma - \ln^2 x_\sigma - \Upsilon(\frac{x_{iw}}{x_\sigma})) + x_{H_k^-w} \ln x_\sigma + x_{iw} \ln^2(x_{H_k^-w}x_{iw})}{(x_{iw} - x_\sigma)^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \\
& + \frac{2(5x_{iw} - 3x_{H_k^-w})(-1 + \ln x_{iw} + \ln^2 x_{iw}) + x_{H_k^-w}(1 + \ln x_{iw}) + x_{iw}(8 - \ln x_{iw})}{(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \frac{6x_{iw}(x_{H_k^-w} - x_{iw})(\ln x_{iw} - \ln^2 x_{iw}) + x_{iw}(4x_{iw} + x_{H_k^-w}) \ln x_{iw}}{(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})^2} \\
& + \frac{6x_{iw}(x_{H_k^-w} - x_{iw})(\ln x_{iw} - \ln^2 x_{iw}) + x_{iw}(4x_{iw} + x_{H_k^-w}) \ln x_{iw}}{(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})^2(x_{jw} - x_{iw})} \\
& + \frac{6x_{iw}(x_{H_k^-w} - x_{iw})(\ln x_{iw} - \ln^2 x_{iw}) + x_{iw}(4x_{iw} + x_{H_k^-w}) \ln x_{iw}}{(x_{H_k^-w} - x_{iw})^2(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \left( \frac{16x_{H_l^-w} \Upsilon(\frac{x_{iw}}{x_{H_l^-w}}) + x_{iw}(\ln x_{iw} - 2 \ln^2 x_{iw}) - x_{H_l^-w}(13 \ln x_{H_l^-w} - 14 \ln^2 x_{H_l^-w})}{(-x_{H_l^-w} + x_{iw})^2(-x_{H_l^-w} + x_{jw})} + (x_{H_l^-w} \leftrightarrow x_{jw}) \right) \\
& + 16 \left( \frac{-1 - \ln x_{iw} - \ln^2 x_{iw}}{(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})} + \frac{x_{iw}(\ln x_{iw} - \ln^2 x_{iw})}{(x_{H_l^-w} - x_{iw})(x_{jw} - x_{iw})^2} + \frac{x_{iw}(\ln x_{iw} - \ln^2 x_{iw})}{(x_{H_l^-w} - x_{iw})^2(x_{jw} - x_{iw})} \right. \\
& \left. - \frac{1 - 3 \ln x_{iw} - 2 \ln^2 x_{iw}}{(x_{H_l^-w} - x_{iw})(x_{H_l^-w} - x_{jw})} \right) - 2 \left( \sum_{\sigma=u^i, u^j, H_l^-} \left( \frac{\mathcal{R}_{i2}(\frac{x_{H_k^-w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \right. \\
& \left. \left. - \frac{(x_{H_l^-w} - x_{jw})(\mathcal{R}_{i2}(\frac{x_{H_k^-w}}{x_\sigma}) - \mathcal{R}_{i2}(\frac{x_{jw}}{x_\sigma})) + x_\sigma(\Upsilon(\frac{x_{H_k^-w}}{x_\sigma}) - \Upsilon(\frac{x_{jw}}{x_\sigma}))}{(x_{H_k^-w} - x_{jw}) \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right) \right. \\
& \left. - \frac{\mathcal{R}_{i2}(\frac{x_{H_k^-w}}{x_{iw}}) - \mathcal{R}_{i2}(\frac{x_{H_k^-w}}{x_{jw}}) - \mathcal{R}_{i2}(\frac{x_{jw}}{x_{iw}})}{(x_{H_k^-w} - x_{jw})(x_{iw} - x_{jw})} \right) + \frac{2x_{H_l^-w}(\ln x_{H_l^-w} + \ln^2 x_{H_l^-w})}{(x_{H_k^-w} - x_{H_l^-w})(-x_{H_k^-w} + x_{iw})^2} \\
& + 2 \sum_{\sigma=u^i, u^j, H_k^-, H_l^-} \frac{x_\sigma(-2 \ln x_\sigma + \ln^2 x_\sigma + 2(x_{H_k^-w} - x_{iw})\mathcal{R}_{i2}(\frac{x_{iw}}{x_\sigma}) + 2\Upsilon(\frac{x_{iw}}{x_\sigma}))}{2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \Big] \\
& - \frac{1}{3 \sin^2 \beta} x_{iw} x_{jw} h_b h_d ((\mathcal{Z}_H^{1k})^2 (\mathcal{Z}_H^{2l})^2 + (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{1l})^2) \Big[ \\
& - 8 \left( \sum_{\sigma=u^j, H_k^-, H_l^-} \frac{x_{H_k^-w} x_\sigma (\ln x_\sigma - 2 \ln^2 x_\sigma - 2\Upsilon(\frac{x_{iw}}{x_\sigma}))}{(x_{iw} - x_\sigma)^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \\
& \left. + \left( \frac{-x_{H_l^-w} \ln x_{H_l^-w} + \ln^2 x_{H_l^-w} + 2\Upsilon(\frac{x_{iw}}{x_{H_l^-w}})}{2(-x_{H_l^-w} + x_{iw})^2(-x_{H_l^-w} + x_{jw})} + (x_{H_l^-w} \leftrightarrow x_{jw}) \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{x_{H_k^- w}(-1 + 3 \ln x_{iw} + 2 \ln^2 x_{iw})}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} + \frac{x_{H_k^- w} x_{iw}(-\ln x_{iw} + 2 \ln^2 x_{iw})}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})^2} \\
& + \frac{x_{H_k^- w} x_{iw}(-\ln x_{iw} + 2 \ln^2 x_{iw})}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})^2(x_{jw} - x_{iw})} + \frac{x_{H_k^- w} x_{iw}(-\ln x_{iw} + 2 \ln^2 x_{iw})}{(x_{H_k^- w} - x_{iw})^2(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} \\
& - \frac{x_{iw}(\ln x_{iw} - 2 \ln^2 x_{iw})}{(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})^2} - \frac{x_{iw}(\ln x_{iw} - 2 \ln^2 x_{iw})}{(x_{H_l^- w} - x_{iw})^2(x_{jw} - x_{iw})} - \frac{1 - 3 \ln x_{iw} - 2 \ln^2 x_{iw}}{(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} \Big) \\
& + \sum_{\sigma=u^i, u^j, H_l^-} \left( \frac{\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + \frac{(x_{H_l^- w} - x_{jw})(\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_\sigma}) - \mathcal{R}_{i_2}(\frac{x_{jw}}{x_\sigma})) + x_\sigma(\Upsilon(\frac{x_{H_k^- w}}{x_\sigma}) - \Upsilon(\frac{x_{jw}}{x_\sigma}))}{(x_{H_k^- w} - x_{jw}) \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right) \\
& - \frac{\mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_{iw}}) - \mathcal{R}_{i_2}(\frac{x_{H_k^- w}}{x_{jw}}) - \mathcal{R}_{i_2}(\frac{x_{jw}}{x_{iw}})}{(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} \Big) - 16 \sum_{\sigma=u^i, H_k^-, H_l^-} \frac{\mathcal{R}_{i_2}(\frac{x_{jw}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + 48 \sum_{\sigma=u^i, u^j, H_l^-} \frac{x_\sigma \mathcal{R}_{i_2}(\frac{x_{iw}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \\
& - 8 \left( \sum_{\sigma=u^i, u^j, H_k^-, H_l^-} \frac{13x_\sigma \ln^2 x_\sigma - 2x_\sigma \ln x_\sigma - 6x_\sigma(x_{H_k^- w} - x_{iw})\mathcal{R}_{i_2}(\frac{x_{iw}}{x_\sigma}) + 10x_\sigma \Upsilon(\frac{x_{iw}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \\
& + \frac{x_{H_l^- w}(2 \ln x_{H_l^- w} + 3 \ln^2 x_{H_l^- w})}{(x_{H_k^- w} - x_{H_l^- w})(-x_{H_l^- w} + x_{iw})(-x_{H_l^- w} + x_{jw})} - \frac{x_{H_l^- w}(2 \ln x_{H_l^- w} + 3 \ln^2 x_{H_l^- w})}{(x_{H_k^- w} - x_{H_l^- w})(-x_{H_l^- w} + x_{iw})^2} \Big) \\
& - 16 \left( \sum_{\sigma=u^j, H_k^-, H_l^-} \frac{x_\sigma(2x_{iw} \ln x_\sigma - 2(x_{H_k^- w} + x_{iw})(\ln^2 x_\sigma + \Upsilon(\frac{x_{iw}}{x_\sigma})) - x_{iw} \ln^2(x_{iw} x_{jw}))}{2(x_{iw} - x_\sigma)^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \\
& + \frac{-x_{iw} + (2x_{H_k^- w} + 4x_{iw}) \ln x_{iw} + (x_{H_k^- w} + 5x_{iw}) \ln^2 x_{iw}}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} + \frac{x_{iw}^2 \ln x_{iw} + x_{iw}(x_{H_k^- w} + 3x_{iw}) \ln^2 x_{iw}}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})^2} \\
& + \frac{x_{iw}^2 \ln x_{iw} + x_{iw}(x_{H_k^- w} + 3x_{iw}) \ln^2 x_{iw}}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})^2(x_{jw} - x_{iw})} + \frac{x_{iw}^2 \ln x_{iw} + x_{iw}(x_{H_k^- w} + 3x_{iw}) \ln^2 x_{iw}}{(x_{H_k^- w} - x_{iw})^2(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \left( \frac{x_{H_l^- w}(\ln^2 x_{H_l^- w} + \Upsilon(\frac{x_{iw}}{x_{H_l^- w}}))}{(-x_{H_l^- w} + x_{iw})^2(-x_{H_l^- w} + x_{jw})} + (x_{H_l^- w} \leftrightarrow x_{jw}) \right) - \frac{2 \ln x_{iw} + \ln^2 x_{iw}}{(-x_{H_l^- w} + x_{iw})(-x_{iw} + x_{jw})} \\
& \left. - \frac{x_{iw} \ln^2 x_{iw}}{(-x_{H_l^- w} + x_{iw})(-x_{iw} + x_{jw})^2} - \frac{x_{iw} \ln^2 x_{iw}}{(-x_{H_l^- w} + x_{iw})^2(-x_{iw} + x_{jw})} \right) \Big], \tag{77}
\end{aligned}$$

$$HH_3 = -2HH_2, \tag{78}$$

$$\begin{aligned}
HH_4 &= \frac{h_b^2 + h_d^2}{\sin^2 \beta} x_{iw} x_{jw} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \left[ \left( -F_A^{1a} - F_A^{1b} + \frac{13}{32} F_A^{1c} \right) (x_{iw}, x_{jw}, x_{H_l^- w}, 0, x_{iw}, x_{jw}, x_{H_k^- w}) \right. \\
& \left. - \frac{4}{3} \left( - \sum_{\sigma=u^j, H_k^-, H_l^-} \frac{x_{H_k^- w} x_\sigma (\ln x_\sigma - 2 \ln^2 x_\sigma - 2 \Upsilon(\frac{x_{iw}}{x_\sigma}))}{(x_{iw} - x_\sigma)^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + \left( \frac{-x_{H_l^- w} \ln x_{H_l^- w} + \ln^2 x_{H_l^- w} + 2\Upsilon\left(\frac{x_{iw}}{x_{H_l^- w}}\right)}{(-x_{H_l^- w} + x_{iw})^2(-x_{H_l^- w} + x_{jw})} + (x_{H_l^- w} \leftrightarrow x_{jw}) \right) \\
& + \frac{x_{H_k^- w}(3 \ln x_{iw} + 2 \ln^2 x_{iw})}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} + \frac{x_{H_k^- w} x_{iw}(-\ln x_{iw} + 2 \ln^2 x_{iw})}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})^2} \\
& + \frac{x_{H_k^- w} x_{iw}(-\ln x_{iw} + 2 \ln^2 x_{iw})}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})^2(x_{jw} - x_{iw})} + \frac{x_{H_k^- w} x_{iw}(-\ln x_{iw} + 2 \ln^2 x_{iw})}{(x_{H_k^- w} - x_{iw})^2(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \frac{x_{iw}(\ln x_{iw} - 2 \ln^2 x_{iw})}{(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})^2} + \frac{x_{iw}(\ln x_{iw} - 2 \ln^2 x_{iw})}{(x_{H_l^- w} - x_{iw})^2(x_{jw} - x_{iw})} + \frac{1 - 3 \ln x_{iw} - 2 \ln^2 x_{iw}}{(x_{H_l^- w} - x_{iw})(x_{jw} - x_{iw})} \Big) \\
& - \frac{4}{3} \left( - \sum_{\sigma=u^i, u^j, H_l^-} \frac{3x_\sigma \mathcal{R}_{i_2}\left(\frac{x_{iw}}{x_\sigma}\right)}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + \frac{x_{H_l^- w}(2 \ln x_{H_l^- w} + 3 \ln^2 x_{H_l^- w})}{2(x_{H_k^- w} - x_{H_l^- w})(-x_{H_l^- w} + x_{iw})(-x_{H_l^- w} + x_{jw})} \right. \\
& + \sum_{\sigma=u^i, u^j, H_k^-, H_l^-} \frac{x_\sigma \ln x_\sigma - 13x_\sigma \ln^2 x_\sigma + 6x_\sigma(x_{H_k^- w} - x_{iw})\mathcal{R}_{i_2}\left(\frac{x_{iw}}{x_\sigma}\right) - 10x_\sigma \Upsilon\left(\frac{x_{iw}}{x_\sigma}\right)}{2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \\
& \left. - \frac{x_{H_l^- w}(2 \ln x_{H_l^- w} + 3 \ln^2 x_{H_l^- w})}{2(x_{H_k^- w} - x_{H_l^- w})(-x_{H_l^- w} + x_{iw})^2} \right) + \frac{7}{6} \left( \frac{\mathcal{R}_{i_2}\left(\frac{x_{H_k^- w}}{x_{iw}}\right) - \mathcal{R}_{i_2}\left(\frac{x_{H_k^- w}}{x_{jw}}\right) - \mathcal{R}_{i_2}\left(\frac{x_{jw}}{x_{iw}}\right)}{(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} \right. \\
& - \sum_{\sigma=u^i, u^j, H_l^-} \left( \frac{\mathcal{R}_{i_2}\left(\frac{x_{H_k^- w}}{x_\sigma}\right)}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + \frac{(x_{H_l^- w} - x_{jw})(\mathcal{R}_{i_2}\left(\frac{x_{H_k^- w}}{x_\sigma}\right) - \mathcal{R}_{i_2}\left(\frac{x_{jw}}{x_\sigma}\right)) + x_\sigma(\Upsilon\left(\frac{x_{H_k^- w}}{x_\sigma}\right) - \Upsilon\left(\frac{x_{jw}}{x_\sigma}\right))}{(x_{H_k^- w} - x_{jw}) \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right) \Big) \\
& - \frac{1}{2} \frac{\mathcal{R}_{i_2}\left(\frac{x_{jw}}{x_{iw}}\right) - \mathcal{R}_{i_2}\left(\frac{x_{H_k^- w}}{x_{H_l^- w}}\right) - \mathcal{R}_{i_2}\left(\frac{x_{H_k^- w}}{x_{iw}}\right) - \mathcal{R}_{i_2}\left(\frac{x_{jw}}{x_{H_l^- w}}\right)}{(-x_{H_l^- w} + x_{iw})(-x_{H_k^- w} + x_{jw})} \\
& + \frac{1}{4} \sum_{\sigma=u^i, u^j, H_l^-} \left[ \frac{\mathcal{R}_{i_2}\left(\frac{x_{H_k^- w}}{x_\sigma}\right)}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + (x_{iw} \leftrightarrow x_{jw}) \right], \tag{79}
\end{aligned}$$

$$HH_5 = \frac{1}{4}HH_4, \tag{80}$$

$$\begin{aligned}
cc_1 &= \frac{1}{6}a_+^{(c)}b_-^{(c)}c_+^{(c)}d_-^{(c)} \left( F_A^{2a} + F_A^{2b} + \frac{1}{2}F_A^{2c} + 2F_A^{2d} - 3F_A^{2e} - F_A^{2f} \right) (x_{\kappa_\lambda^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{U}_\beta^j w}, 0, x_{\kappa_\eta^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{U}_\beta^j w}) \\
& + \frac{4}{3}a_+^{(c)}b_-^{(c)}c_+^{(c)}d_-^{(c)} \left( \frac{\mathcal{R}_{i_2}\left(\frac{x_{\kappa_\lambda^- w}}{x_{\tilde{U}_\alpha^i w}}\right) - \mathcal{R}_{i_2}\left(\frac{x_{\kappa_\lambda^- w}}{x_{\tilde{U}_\beta^j w}}\right)}{x_{\tilde{U}_\alpha^i w} - x_{\tilde{U}_\beta^j w}} - (x_{\kappa_\eta^- w} - x_{\kappa_\lambda^- w}) \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-} \frac{\mathcal{R}_{i_2}\left(\frac{x_{\kappa_\lambda^- w}}{x_\sigma}\right)}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right) \\
& + \frac{1}{3}a_+^{(c)}b_-^{(c)}c_+^{(c)}d_-^{(c)} \left( - \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-} \frac{x_\sigma(x_{\kappa_\lambda^- w} \ln^2(x_\sigma x_{\kappa_\lambda^- w}) - x_{\tilde{U}_\beta^j w} \ln^2(x_\sigma x_{\tilde{U}_\beta^j w}))}{2(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\beta^j w}) \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{x_{\tilde{U}_{\beta}^j w}^2 + 2x_{\kappa_{\lambda}^- w} x_{\tilde{U}_{\beta}^j w} - 2x_{\kappa_{\lambda}^- w}^2}{x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\beta}^j w}} \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^i, \kappa_{\eta}^-} \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_{\lambda}^- w}}{x_{\sigma}}) - \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \\
& + (x_{\tilde{U}_{\beta}^j w} + 2x_{\kappa_{\eta}^- w}) \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^i, \kappa_{\eta}^-} \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_{\lambda}^- w}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} + 2 \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_{\eta}^- w}}{x_{\tilde{U}_{\alpha}^i w}}) - \mathcal{R}_{i_2}(\frac{x_{\kappa_{\eta}^- w}}{x_{\tilde{U}_{\beta}^j w}})}{x_{\tilde{U}_{\alpha}^i w} - x_{\tilde{U}_{\beta}^j w}} \\
& + 2 \frac{\mathbb{E}(x_{\tilde{U}_{\alpha}^i w}, x_{\kappa_{\eta}^- w}) - \mathbb{E}(x_{\tilde{U}_{\alpha}^i w}, x_{\tilde{U}_{\beta}^j w})}{x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\beta}^j w}} + \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^i, \kappa_{\eta}^-} \frac{x_{\sigma} \Upsilon(\frac{x_{\kappa_{\lambda}^- w}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \\
& + 3 \frac{x_{\tilde{U}_{\alpha}^i w} \Upsilon(\frac{x_{\kappa_{\lambda}^- w}}{x_{\tilde{U}_{\beta}^j w}}) - x_{\tilde{U}_{\beta}^j w} \Upsilon(\frac{x_{\kappa_{\lambda}^- w}}{x_{\tilde{U}_{\beta}^j w}})}{(x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\beta}^j w})(x_{\tilde{U}_{\alpha}^i w} - x_{\tilde{U}_{\beta}^j w})} + \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^i, \kappa_{\eta}^-} \frac{x_{\kappa_{\eta}^- w} x_{\sigma} (\Upsilon(\frac{x_{\kappa_{\lambda}^- w}}{x_{\sigma}}) - \Upsilon(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\sigma}}))}{(x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\beta}^j w}) \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \\
& + \frac{16}{3} a_+^{(c)} b_-^{(c)} c_+^{(c)} d_+^{(c)} \left( -2(x_{\tilde{U}_{\alpha}^i w} - x_{\tilde{U}_{\beta}^j w}) \sum_{\sigma=\tilde{U}_{\alpha}^i, \kappa_{\eta}^-, \kappa_{\lambda}^-} \frac{\mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \right. \\
& + \sum_{\sigma=\tilde{U}_{\alpha}^i, \kappa_{\eta}^-, \kappa_{\lambda}^-} \frac{x_{\sigma} (2 \ln x_{\sigma} - 3 \ln^2 x_{\sigma} - 2 \Upsilon(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\sigma}}))}{2 \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} + 2 \frac{\mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\kappa_{\eta}^- w}}) - \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\kappa_{\lambda}^- w}})}{-x_{\kappa_{\eta}^- w} + x_{\kappa_{\lambda}^- w}} \\
& \left. - \frac{x_{\kappa_{\eta}^- w} \ln^2 x_{\kappa_{\eta}^- w}}{(-x_{\kappa_{\eta}^- w} + x_{\kappa_{\lambda}^- w})(-x_{\kappa_{\eta}^- w} + x_{\tilde{U}_{\alpha}^i w})} - \frac{x_{\tilde{U}_{\alpha}^i w} \ln^2 x_{\tilde{U}_{\alpha}^i w}}{(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha}^i w})(x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\alpha}^i w})} \right) \\
& - \frac{2}{3} a_+^{(c)} b_-^{(c)} c_+^{(c)} d_-^{(c)} \left( - \frac{-\mathbb{E}(x_{\tilde{U}_{\alpha}^i w}, x_{\kappa_{\lambda}^- w}) + \mathbb{E}(x_{\tilde{U}_{\alpha}^i w}, x_{\tilde{U}_{\beta}^j w})}{(-x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\beta}^j w})} + \frac{-\mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\kappa_{\eta}^- w}}) + \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\tilde{U}_{\alpha}^i w}})}{-x_{\kappa_{\eta}^- w} + x_{\tilde{U}_{\alpha}^i w}} \right. \\
& + \frac{x_{\kappa_{\eta}^- w} + x_{\kappa_{\lambda}^- w}}{(-x_{\kappa_{\eta}^- w} + x_{\tilde{U}_{\alpha}^i w})(-x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\beta}^j w})} \left( -\mathcal{R}_{i_2}(\frac{x_{\kappa_{\lambda}^- w}}{x_{\kappa_{\eta}^- w}}) - \mathcal{R}_{i_2}(\frac{x_{\kappa_{\lambda}^- w}}{x_{\tilde{U}_{\alpha}^i w}}) - \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\kappa_{\eta}^- w}}) + \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\tilde{U}_{\alpha}^i w}}) \right) \\
& \left. - \frac{1}{(-x_{\kappa_{\eta}^- w} + x_{\tilde{U}_{\alpha}^i w})(-x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\beta}^j w})} \left( x_{\kappa_{\eta}^- w} (\Upsilon(\frac{x_{\kappa_{\lambda}^- w}}{x_{\kappa_{\eta}^- w}}) - \Upsilon(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\kappa_{\eta}^- w}})) - x_{\tilde{U}_{\alpha}^i w} (\Upsilon(\frac{x_{\kappa_{\lambda}^- w}}{x_{\tilde{U}_{\alpha}^i w}}) - \Upsilon(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\tilde{U}_{\alpha}^i w}})) \right) \right) \\
& + \frac{16}{3} a_+^{(c)} b_-^{(c)} c_+^{(c)} d_-^{(c)} \left( \sum_{\sigma=\tilde{U}_{\alpha}^i, \kappa_{\eta}^-, \kappa_{\lambda}^-} \left( \frac{x_{\tilde{U}_{\alpha}^i w} x_{\sigma} (-2 \ln x_{\sigma} - 16 \ln^2 x_{\sigma} - 3 \ln^2 (x_{\sigma} x_{\tilde{U}_{\alpha}^i w}))}{4(x_{\sigma} - x_{\tilde{U}_{\alpha}^i w})^2 \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \right. \right. \\
& \left. + \frac{x_{\tilde{U}_{\alpha}^i w} x_{\sigma} (3 \ln x_{\sigma} + 12 \ln^2 x_{\sigma} - \ln^2 (x_{\sigma} x_{\tilde{U}_{\alpha}^i w}))}{2(x_{\sigma} - x_{\tilde{U}_{\alpha}^i w}) \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} - \frac{2x_{\sigma} \ln^2 x_{\sigma}}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \right)
\end{aligned}$$



$$\begin{aligned}
& + \frac{x_{\tilde{U}_{\alpha}^i w}^3 (\ln x_{\tilde{U}_{\alpha}^i w} + 14 \ln^2 x_{\tilde{U}_{\alpha}^i w})}{2(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha}^i w})(x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\alpha}^i w})(x_{\tilde{U}_{\beta}^j w} - x_{\tilde{U}_{\alpha}^i w})^2} + \frac{x_{\tilde{U}_{\alpha}^i w}^2 (1 + 29 \ln x_{\tilde{U}_{\alpha}^i w} + 28 \ln^2 x_{\tilde{U}_{\alpha}^i w})}{2(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha}^i w})(x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\alpha}^i w})(x_{\tilde{U}_{\beta}^j w} - x_{\tilde{U}_{\alpha}^i w})} \\
& + \frac{x_{\tilde{U}_{\alpha}^i w}^3 (\ln x_{\tilde{U}_{\alpha}^i w} + 14 \ln^2 x_{\tilde{U}_{\alpha}^i w})}{2(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha}^i w})(x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\alpha}^i w})^2 (x_{\tilde{U}_{\beta}^j w} - x_{\tilde{U}_{\alpha}^i w})} + \frac{x_{\tilde{U}_{\alpha}^i w}^3 (\ln x_{\tilde{U}_{\alpha}^i w} + 14 \ln^2 x_{\tilde{U}_{\alpha}^i w})}{2(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha}^i w})^2 (x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\alpha}^i w})(x_{\tilde{U}_{\beta}^j w} - x_{\tilde{U}_{\alpha}^i w})} \\
& - \frac{x_{\kappa_{\eta}^- w} \ln x_{\kappa_{\eta}^- w}}{(x_{\kappa_{\lambda}^- w} - x_{\kappa_{\eta}^- w})(x_{\tilde{U}_{\beta}^j w} - x_{\kappa_{\eta}^- w})} + 2 \frac{x_{\kappa_{\lambda}^- w} \ln x_{\kappa_{\lambda}^- w}}{(x_{\kappa_{\eta}^- w} - x_{\kappa_{\lambda}^- w})(x_{\tilde{U}_{\beta}^j w} - x_{\kappa_{\lambda}^- w})} - \frac{3x_{\kappa_{\lambda}^- w} \ln x_{\kappa_{\lambda}^- w}}{(x_{\kappa_{\eta}^- w} - x_{\kappa_{\lambda}^- w})^2} \\
& + \frac{16}{3} a_+^{(c)} b_-^{(c)} c_+^{(c)} d_-^{(c)} \left( \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^j, \kappa_{\eta}^-, \kappa_{\lambda}^-} \frac{x_{\kappa_{\lambda}^- w} x_{\sigma} (6 \ln x_{\sigma} - 7 \ln^2 x_{\sigma})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} + \sum_{\sigma=\tilde{U}_{\alpha}^i, \kappa_{\eta}^-, \kappa_{\lambda}^-} \frac{x_{\sigma} (3 \ln x_{\sigma} + 7 \ln^2 x_{\sigma})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \right. \\
& - \frac{x_{\kappa_{\eta}^- w} x_{\kappa_{\lambda}^- w} (3 \ln x_{\kappa_{\eta}^- w} + \ln^2 x_{\kappa_{\eta}^- w})}{2(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha}^i w})(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha}^i w})^2} + \frac{4x_{\kappa_{\eta}^- w} x_{\kappa_{\lambda}^- w} \ln^2 x_{\kappa_{\eta}^- w}}{2(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha}^i w})(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha}^i w})(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\beta}^j w})} \\
& + \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^j, \kappa_{\eta}^-, \kappa_{\lambda}^-} \left( \frac{(x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\beta}^j w}) x_{\kappa_{\lambda}^- w} x_{\sigma} \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} - \frac{3x_{\kappa_{\lambda}^- w} x_{\sigma} \Upsilon(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \right) \\
& + \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^j, \kappa_{\eta}^-} \left( \frac{(x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\beta}^j w} + x_{\kappa_{\lambda}^- w}) x_{\sigma} \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} + \frac{3x_{\sigma} \Upsilon(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} + \frac{\mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\tilde{U}_{\alpha}^i w}})}{x_{\tilde{U}_{\alpha}^i w} - x_{\tilde{U}_{\beta}^j w}} \right), \tag{81}
\end{aligned}$$

$$\begin{aligned}
cc_2 = & \frac{1}{24} (a_-^{(c)} b_+^{(c)} c_+^{(c)} d_-^{(c)} + a_+^{(c)} b_-^{(c)} c_-^{(c)} d_+^{(c)}) (F_A^{2a} + F_A^{2b} + \frac{3}{2} F_A^{2c} + 6 F_A^{2d} \\
& - \frac{5}{2} F_A^{2e} + \frac{1}{2} F_A^{2f}) (x_{\kappa_{\lambda}^- w}, x_{\tilde{U}_{\alpha}^i w}, x_{\tilde{U}_{\beta}^j w}, 0, x_{\kappa_{\eta}^- w}, x_{\tilde{U}_{\alpha}^i w}, x_{\tilde{U}_{\beta}^j w}) \\
& + (a_-^{(c)} b_+^{(c)} c_+^{(c)} d_-^{(c)} + a_+^{(c)} b_-^{(c)} c_-^{(c)} d_+^{(c)}) \left[ \left( \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_{\lambda}^- w}}{x_{\tilde{U}_{\alpha}^i w}}) - \mathcal{R}_{i_2}(\frac{x_{\kappa_{\lambda}^- w}}{x_{\tilde{U}_{\beta}^j w}})}{x_{\tilde{U}_{\alpha}^i w} - x_{\tilde{U}_{\beta}^j w}} + \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^j, \kappa_{\eta}^-} \frac{4(x_{\sigma} - x_{\kappa_{\lambda}^- w}) \mathcal{R}_{i_2}(\frac{x_{\kappa_{\lambda}^- w}}{x_{\sigma}})}{3 \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \right. \right. \\
& \left. \left. + \frac{7}{48} \frac{-x_{\kappa_{\eta}^- w} (-\Upsilon(\frac{x_{\kappa_{\lambda}^- w}}{x_{\kappa_{\eta}^- w}}) + \Upsilon(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\kappa_{\eta}^- w}})) + x_{\tilde{U}_{\alpha}^i w} (-\Upsilon(\frac{x_{\kappa_{\lambda}^- w}}{x_{\tilde{U}_{\alpha}^i w}}) + \Upsilon(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\tilde{U}_{\alpha}^i w}}))}{(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha}^i w})(x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\beta}^j w})} \right) \\
& - \frac{1}{48} \left( - \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^j, \kappa_{\eta}^-} \frac{x_{\sigma} (x_{\kappa_{\lambda}^- w} \ln^2(x_{\sigma} x_{\kappa_{\lambda}^- w}) - x_{\tilde{U}_{\beta}^j w} \ln^2(x_{\sigma} x_{\tilde{U}_{\beta}^j w}))}{2(x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\beta}^j w}) \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \right. \\
& \left. + \frac{x_{\tilde{U}_{\beta}^j w}^2 + 2x_{\kappa_{\lambda}^- w} x_{\tilde{U}_{\beta}^j w} - 2x_{\kappa_{\lambda}^- w}^2}{x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\beta}^j w}} \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^j, \kappa_{\eta}^-} \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_{\lambda}^- w}}{x_{\sigma}}) - \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j w}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} + 2 \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_{\eta}^- w}}{x_{\tilde{U}_{\alpha}^i w}}) - \mathcal{R}_{i_2}(\frac{x_{\kappa_{\eta}^- w}}{x_{\tilde{U}_{\beta}^j w}})}{x_{\tilde{U}_{\alpha}^i w} - x_{\tilde{U}_{\beta}^j w}} \right)
\end{aligned}$$

$$\begin{aligned}
& + (x_{\tilde{U}_{\beta}^j} + 2x_{\kappa_{\eta}^-}) \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^i, \kappa_{\eta}^-} \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_{\lambda}^-}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} + 2 \frac{\mathbb{E}(x_{\tilde{U}_{\alpha}^i}, x_{\kappa_{\eta}^-}) - \mathbb{E}(x_{\tilde{U}_{\alpha}^i}, x_{\tilde{U}_{\beta}^j})}{x_{\kappa_{\lambda}^-} - x_{\tilde{U}_{\beta}^j}} \\
& + 3 \frac{x_{\tilde{U}_{\alpha}^i} \Upsilon(\frac{x_{\kappa_{\lambda}^-}}{x_{\tilde{U}_{\beta}^j}}) - x_{\tilde{U}_{\beta}^j} \Upsilon(\frac{x_{\kappa_{\lambda}^-}}{x_{\tilde{U}_{\beta}^j}})}{(x_{\kappa_{\lambda}^-} - x_{\tilde{U}_{\beta}^j})(x_{\tilde{U}_{\alpha}^i} - x_{\tilde{U}_{\beta}^j})} + \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^i, \kappa_{\eta}^-} \frac{x_{\kappa_{\eta}^-} x_{\sigma} (\Upsilon(\frac{x_{\kappa_{\lambda}^-}}{x_{\sigma}}) - \Upsilon(\frac{x_{\tilde{U}_{\beta}^j}}{x_{\sigma}}))}{(x_{\kappa_{\lambda}^-} - x_{\tilde{U}_{\beta}^j}) \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \\
& + \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^i, \kappa_{\eta}^-} \frac{x_{\sigma} \Upsilon(\frac{x_{\kappa_{\lambda}^-}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} - \frac{8}{3} \left( \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^j, \kappa_{\eta}^-} \frac{x_{\sigma} (-10 \ln x_{\sigma} + 7 \ln^2 x_{\sigma})}{2 \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \right. \\
& + \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^i, \kappa_{\lambda}^-} \frac{x_{\sigma} (10 \ln x_{\sigma} - 7 \ln^2 x_{\sigma})}{2 \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} + \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^i, \kappa_{\eta}^-, \kappa_{\lambda}^-} \frac{(x_{\kappa_{\lambda}^-}^2 - x_{\tilde{U}_{\beta}^j} x_{\kappa_{\lambda}^-}) x_{\sigma} \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \\
& - \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^j} \frac{(x_{\kappa_{\eta}^-} + x_{\kappa_{\lambda}^-} - x_{\tilde{U}_{\beta}^j}) x_{\sigma} \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} + \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^j, \kappa_{\eta}^-, \kappa_{\lambda}^-} \frac{3 x_{\kappa_{\lambda}^-} x_{\sigma} \Upsilon(\frac{x_{\tilde{U}_{\beta}^j}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \\
& + \sum_{\sigma=\tilde{U}_{\alpha}^i, \tilde{U}_{\beta}^j, \kappa_{\eta}^-} \frac{3 x_{\sigma} \Upsilon(\frac{x_{\tilde{U}_{\beta}^j}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} + \frac{\mathcal{R}_{i_2}(\frac{x_{\tilde{U}_{\beta}^j}}{x_{\tilde{U}_{\alpha}^i}})}{x_{\tilde{U}_{\alpha}^i} - x_{\tilde{U}_{\beta}^j}} + \frac{x_{\kappa_{\eta}^-} x_{\kappa_{\lambda}^-} (\ln x_{\kappa_{\eta}^-} - \ln^2 x_{\kappa_{\eta}^-})}{2(-x_{\kappa_{\eta}^-} + x_{\kappa_{\lambda}^-})(-x_{\kappa_{\eta}^-} + x_{\tilde{U}_{\alpha}^i})^2} \\
& + \frac{x_{\kappa_{\eta}^-} x_{\kappa_{\lambda}^-} (9 \ln x_{\kappa_{\eta}^-} - 6 \ln^2 x_{\kappa_{\eta}^-})}{2(-x_{\kappa_{\eta}^-} + x_{\kappa_{\lambda}^-})(-x_{\kappa_{\eta}^-} + x_{\tilde{U}_{\alpha}^i})(-x_{\kappa_{\eta}^-} + x_{\tilde{U}_{\beta}^j})} + \sum_{\sigma=\tilde{U}_{\beta}^j, \kappa_{\eta}^-, \kappa_{\lambda}^-} \left( \frac{x_{\sigma} (3 \ln x_{\sigma} - 2 \ln^2 x_{\sigma})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \right. \\
& + \frac{x_{\tilde{U}_{\alpha}^i}^2 x_{\sigma} (22 \ln x_{\sigma} - 16 \ln^2 x_{\sigma} + \ln^2(x_{\tilde{U}_{\alpha}^i} x_{\sigma}))}{4(-x_{\sigma} + x_{\tilde{U}_{\alpha}^i})^2 \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} - \frac{x_{\tilde{U}_{\alpha}^i}^2 x_{\sigma} (17 \ln x_{\sigma} - 12 \ln^2 x_{\sigma} + \ln^2(x_{\tilde{U}_{\alpha}^i} x_{\sigma}))}{2(-x_{\sigma} + x_{\tilde{U}_{\alpha}^i}) \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \Big) \\
& - \frac{15 x_{\tilde{U}_{\alpha}^i}^2 \ln x_{\tilde{U}_{\alpha}^i} - 6 x_{\tilde{U}_{\alpha}^i}^2 \ln^2 x_{\tilde{U}_{\alpha}^i} + 11 x_{\tilde{U}_{\alpha}^i}^2}{2(x_{\kappa_{\eta}^-} - x_{\tilde{U}_{\alpha}^i})(x_{\kappa_{\lambda}^-} - x_{\tilde{U}_{\alpha}^i})(x_{\tilde{U}_{\beta}^j} - x_{\tilde{U}_{\alpha}^i})} - \frac{11 x_{\tilde{U}_{\alpha}^i}^2 \ln x_{\tilde{U}_{\alpha}^i} - 6 x_{\tilde{U}_{\alpha}^i}^3 \ln^2 x_{\tilde{U}_{\alpha}^i}}{2(x_{\kappa_{\eta}^-} - x_{\tilde{U}_{\alpha}^i})(x_{\kappa_{\lambda}^-} - x_{\tilde{U}_{\alpha}^i})(x_{\tilde{U}_{\beta}^j} - x_{\tilde{U}_{\alpha}^i})^2} \\
& + \frac{x_{\tilde{U}_{\alpha}^i}^3 (6 \ln^2 x_{\tilde{U}_{\alpha}^i} - 11 \ln x_{\tilde{U}_{\alpha}^i})}{2(x_{\kappa_{\eta}^-} - x_{\tilde{U}_{\alpha}^i})(x_{\kappa_{\lambda}^-} - x_{\tilde{U}_{\alpha}^i})^2 (x_{\tilde{U}_{\beta}^j} - x_{\tilde{U}_{\alpha}^i})} + \frac{x_{\tilde{U}_{\alpha}^i}^3 (6 \ln^2 x_{\tilde{U}_{\alpha}^i} - 11 \ln x_{\tilde{U}_{\alpha}^i})}{2(x_{\kappa_{\eta}^-} - x_{\tilde{U}_{\alpha}^i})^2 (x_{\kappa_{\lambda}^-} - x_{\tilde{U}_{\alpha}^i})(x_{\tilde{U}_{\beta}^j} - x_{\tilde{U}_{\alpha}^i})} \\
& + \frac{x_{\kappa_{\lambda}^-} \ln x_{\kappa_{\lambda}^-}}{(x_{\kappa_{\eta}^-} - x_{\kappa_{\lambda}^-})^2} - \frac{x_{\kappa_{\lambda}^-} \ln x_{\kappa_{\lambda}^-}}{(x_{\kappa_{\eta}^-} - x_{\kappa_{\lambda}^-})(x_{\tilde{U}_{\beta}^j} - x_{\kappa_{\lambda}^-})} - \sum_{\sigma=\tilde{U}_{\beta}^j, \kappa_{\eta}^-, \kappa_{\lambda}^-} \left( \frac{2 x_{\sigma} \Upsilon(\frac{x_{\tilde{U}_{\alpha}^i}}{x_{\sigma}})}{\prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \right. \\
& + \left. \left. \frac{4 x_{\tilde{U}_{\alpha}^i}^4 x_{\sigma} \Upsilon(\frac{x_{\tilde{U}_{\alpha}^i}}{x_{\sigma}})}{(-x_{\sigma} + x_{\tilde{U}_{\alpha}^i})^2 \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} - \frac{6 x_{\tilde{U}_{\alpha}^i} x_{\sigma} \Upsilon(\frac{x_{\tilde{U}_{\alpha}^i}}{x_{\sigma}})}{(-x_{\sigma} + x_{\tilde{U}_{\alpha}^i}) \prod_{\rho \neq \sigma} (x_{\rho} - x_{\sigma})} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + \left( a_+^{(c)} b_+^{(c)} c_-^{(c)} d_-^{(c)} + a_-^{(c)} b_-^{(c)} c_+^{(c)} d_+^{(c)} \right) \sqrt{x_{\kappa_\eta^- w} x_{\kappa_\lambda^- w}} \left[ \frac{4}{3} \left( - \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-} \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_\lambda^- w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \right. \\
& + \frac{x_{\kappa_\eta^- w} - x_{\tilde{U}_\beta^j w}}{x_{\kappa_\lambda^- w} - x_{\tilde{U}_\beta^j w}} \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-} \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_\lambda^- w}}{x_\sigma}) - \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_\beta^j w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_\lambda^- w}}{x_{\tilde{U}_\alpha^i w}}) - \mathcal{R}_{i_2}(\frac{x_{\kappa_\lambda^- w}}{x_{\tilde{U}_\beta^j w}}) - \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_\beta^j w}}{x_{\tilde{U}_\alpha^i w}})}{(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\beta^j w})(x_{\tilde{U}_\alpha^i w} - x_{\tilde{U}_\beta^j w})} \Big) \\
& - \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_\lambda^- w}}{x_{\kappa_\eta^- w}}) + \mathcal{R}_{i_2}(\frac{x_{\kappa_\lambda^- w}}{x_{\tilde{U}_\alpha^i w}}) + \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_\alpha^i w}}{x_{\kappa_\eta^- w}}) - \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_\beta^j w}}{x_{\tilde{U}_\alpha^i w}})}{2(x_{\tilde{U}_\alpha^i w} - x_{\kappa_\eta^- w})(x_{\tilde{U}_\beta^j w} - x_{\kappa_\lambda^- w})} + \frac{28}{3} \left( \sum_{\sigma=\tilde{U}_\beta^j, \kappa_\eta^-, \kappa_\lambda^-} \left( \frac{2x_{\tilde{U}_\alpha^i w}(x_\sigma \ln x_\sigma - \ln^2 x_\sigma)}{(x_{\tilde{U}_\alpha^i w} - x_\sigma) \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \right. \\
& - \frac{x_\sigma(2 \ln x_\sigma - \ln^2 x_\sigma)}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} - \frac{2x_\sigma x_{\tilde{U}_\alpha^i w} \Upsilon(\frac{x_{\tilde{U}_\alpha^i w}}{x_\sigma})}{(x_{\tilde{U}_\alpha^i w} - x_\sigma) \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + \frac{x_\sigma \Upsilon(\frac{x_{\tilde{U}_\alpha^i w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \Big) \\
& + \frac{-2x_{\tilde{U}_\alpha^i w} + x_{\tilde{U}_\alpha^i w} \ln x_{\tilde{U}_\alpha^i w} + 3x_{\tilde{U}_\alpha^i w} \ln^2 x_{\tilde{U}_\alpha^i w}}{(x_{\kappa_\eta^- w} - x_{\tilde{U}_\alpha^i w})(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\alpha^i w})(x_{\tilde{U}_\beta^j w} - x_{\tilde{U}_\alpha^i w})} + \frac{-2x_{\tilde{U}_\alpha^i w}^2 (\ln x_{\tilde{U}_\alpha^i w} + \ln^2 x_{\tilde{U}_\alpha^i w})}{(x_{\kappa_\eta^- w} - x_{\tilde{U}_\alpha^i w})(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\alpha^i w})(x_{\tilde{U}_\beta^j w} - x_{\tilde{U}_\alpha^i w})^2} \\
& + \frac{-2x_{\tilde{U}_\alpha^i w}^2 (\ln x_{\tilde{U}_\alpha^i w} + \ln^2 x_{\tilde{U}_\alpha^i w})}{(x_{\kappa_\eta^- w} - x_{\tilde{U}_\alpha^i w})(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\alpha^i w})^2 (x_{\tilde{U}_\beta^j w} - x_{\tilde{U}_\alpha^i w})} + \frac{-2x_{\tilde{U}_\alpha^i w}^2 (\ln x_{\tilde{U}_\alpha^i w} + \ln^2 x_{\tilde{U}_\alpha^i w})}{(x_{\kappa_\eta^- w} - x_{\tilde{U}_\alpha^i w})^2 (x_{\kappa_\lambda^- w} - x_{\tilde{U}_\alpha^i w})(x_{\tilde{U}_\beta^j w} - x_{\tilde{U}_\alpha^i w})} \Big) \\
& - \frac{16}{3} \left( - \frac{x_{\kappa_\eta^- w} \ln^2 x_{\kappa_\eta^- w}}{2(-x_{\kappa_\eta^- w} + x_{\kappa_\lambda^- w})(-x_{\kappa_\eta^- w} + x_{\tilde{U}_\alpha^i w})^2} + \frac{4x_{\kappa_\eta^- w} \ln^2 x_{\kappa_\eta^- w}}{(-x_{\kappa_\eta^- w} + x_{\kappa_\lambda^- w})(-x_{\kappa_\eta^- w} + x_{\tilde{U}_\alpha^i w})(-x_{\kappa_\eta^- w} + x_{\tilde{U}_\beta^j w})} \right. \\
& + \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-, \kappa_\lambda^-} \left( \frac{x_\sigma(6 \ln x_\sigma - 7 \ln^2 x_\sigma)}{2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + \frac{(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\beta^j w})x_\sigma \mathcal{R}_{i_2}(\frac{\tilde{U}_\beta^j}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} - 3 \frac{x_\sigma \Upsilon(\frac{\tilde{U}_\beta^j}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right) \\
& \left. - \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-} \frac{x_\sigma \mathcal{R}_{i_2}(\frac{\tilde{U}_\beta^j}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right) \Big], \tag{82}
\end{aligned}$$

$$cc_3 = -2cc_2, \tag{83}$$

$$\begin{aligned}
cc_4 &= a_-^{(c)} b_-^{(c)} c_-^{(c)} d_-^{(c)} \sqrt{x_{\kappa_\eta^- w} x_{\kappa_\lambda^- w}} \left( \frac{4 \left( \mathcal{R}_{i_2}(\frac{x_{\kappa_\lambda^- w}}{x_{\kappa_\eta^- w}}) + \mathcal{R}_{i_2}(\frac{x_{\kappa_\lambda^- w}}{x_{\tilde{U}_\alpha^i w}}) + \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_\beta^j w}}{x_{\kappa_\eta^- w}}) - \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_\beta^j w}}{x_{\tilde{U}_\alpha^i w}}) \right)}{3(-x_{\kappa_\eta^- w} + x_{\tilde{U}_\alpha^i w})(-x_{\kappa_\lambda^- w} + x_{\tilde{U}_\beta^j w})} \right. \\
& - \frac{2}{3} \left( - \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-} \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_\lambda^- w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + \frac{x_{\kappa_\eta^- w} - x_{\tilde{U}_\beta^j w}}{x_{\kappa_\lambda^- w} - x_{\tilde{U}_\beta^j w}} \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-} \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_\lambda^- w}}{x_\sigma}) - \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_\beta^j w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right. \\
& \left. \left. - \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-} \frac{x_\sigma \mathcal{R}_{i_2}(\frac{\tilde{U}_\beta^j}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{\mathcal{R}_{i_2}(\frac{x_{\kappa_\lambda^- w}}{x_{\tilde{U}_\alpha^i w}}) - \mathcal{R}_{i_2}(\frac{x_{\kappa_\lambda^- w}}{x_{\tilde{U}_\beta^j w}}) - \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_\beta^j w}}{x_{\tilde{U}_\alpha^i w}})}{(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\beta^j w})(x_{\tilde{U}_\alpha^i w} - x_{\tilde{U}_\beta^j w})} \Big) - \frac{64}{3} \Big( - \sum_{\sigma=\tilde{U}_\beta^j, \kappa_\eta^-, \kappa_\lambda^-} \frac{2x_{\tilde{U}_\alpha^i w} x_\sigma (\ln x_\sigma - \ln^2 x_\sigma)}{(-x_\sigma + x_{\tilde{U}_\alpha^i w})^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \\
& + \frac{x_{\tilde{U}_\alpha^i w} (2 - \ln x_{\tilde{U}_\alpha^i w} - 3 \ln^2 x_{\tilde{U}_\alpha^i w})}{(x_{\kappa_\eta^- w} - x_{\tilde{U}_\alpha^i w})(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\alpha^i w})(x_{\tilde{U}_\beta^j w} - x_{\tilde{U}_\alpha^i w})} + \frac{2x_{\tilde{U}_\alpha^i w}^2 (\ln x_{\tilde{U}_\alpha^i w} - \ln^2 x_{\tilde{U}_\alpha^i w})}{(x_{\kappa_\eta^- w} - x_{\tilde{U}_\alpha^i w})(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\alpha^i w})(x_{\tilde{U}_\beta^j w} - x_{\tilde{U}_\alpha^i w})^2} \\
& + \frac{2x_{\tilde{U}_\alpha^i w}^2 (\ln x_{\tilde{U}_\alpha^i w} - \ln^2 x_{\tilde{U}_\alpha^i w})}{(x_{\kappa_\eta^- w} - x_{\tilde{U}_\alpha^i w})(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\alpha^i w})^2 (x_{\tilde{U}_\beta^j w} - x_{\tilde{U}_\alpha^i w})} + \frac{2x_{\tilde{U}_\alpha^i w}^2 (\ln x_{\tilde{U}_\alpha^i w} - \ln^2 x_{\tilde{U}_\alpha^i w})}{(x_{\kappa_\eta^- w} - x_{\tilde{U}_\alpha^i w})^2 (x_{\kappa_\lambda^- w} - x_{\tilde{U}_\alpha^i w})(x_{\tilde{U}_\beta^j w} - x_{\tilde{U}_\alpha^i w})} \\
& + \sum_{\sigma=\tilde{U}_\beta^j, \kappa_\eta^-, \kappa_\lambda^-} \frac{(x_{\tilde{U}_\alpha^i w} + x_\sigma) x_\sigma \Upsilon(\frac{x_{\tilde{U}_\alpha^i w}}{x_\sigma})}{(-x_\sigma + x_{\tilde{U}_\alpha^i w})^2 \prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \Big) \\
& + \frac{32}{3} \Big( \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-, \kappa_\lambda^-} \frac{x_\sigma (-3 \ln x_\sigma + 7 \ln^2 x_\sigma)}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + \frac{x_{\kappa_\eta^- w} \ln^2 x_{\kappa_\eta^- w}}{2(-x_{\kappa_\eta^- w} + x_{\kappa_\lambda^- w})(-x_{\kappa_\eta^- w} + x_{\tilde{U}_\alpha^i w})^2} \\
& - \frac{4x_{\kappa_\eta^- w} \ln^2 x_{\kappa_\eta^- w}}{(-x_{\kappa_\eta^- w} + x_{\kappa_\lambda^- w})(-x_{\kappa_\eta^- w} + x_{\tilde{U}_\alpha^i w})(-x_{\kappa_\eta^- w} + x_{\tilde{U}_\beta^j w})} + \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-} \frac{x_\sigma \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_\beta^j w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \\
& + \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-, \kappa_\lambda^-} \frac{x_\sigma (-x_{\kappa_\lambda^- w} + x_{\tilde{U}_\beta^j w}) \mathcal{R}_{i_2}(\frac{x_{\tilde{U}_\beta^j w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} + \sum_{\sigma=\tilde{U}_\alpha^i, \tilde{U}_\beta^j, \kappa_\eta^-, \kappa_\lambda^-} \frac{3x_\sigma \Upsilon(\frac{x_{\tilde{U}_\beta^j w}}{x_\sigma})}{\prod_{\rho \neq \sigma} (x_\rho - x_\sigma)} \Big) \Big) , \tag{84}
\end{aligned}$$

$$cc_5 = \frac{1}{4} cc_4 . \tag{85}$$

In the above expressions Eqs.(81,82,83,84,85), the coupling constants are defined as

$$\begin{aligned}
a_+^{(c)} &= \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1l} + \frac{m_{u^i}}{\sqrt{2}m_w \sin \beta} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2l} \right) , \\
a_-^{(c)} &= \frac{h_d}{\sqrt{2}} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2l} , \\
b_+^{(c)} &= \frac{h_b}{\sqrt{2}} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2l} , \\
b_-^{(c)} &= \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1l} + \frac{m_{u^j}}{\sqrt{2}m_w \sin \beta} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2l} \right) , \\
c_+^{(c)} &= \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1k} + \frac{m_{u^j}}{\sqrt{2}m_w \sin \beta} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2k} \right) , \\
c_-^{(c)} &= \frac{h_d}{\sqrt{2}} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2k} , \\
d_+^{(c)} &= \frac{h_b}{\sqrt{2}} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2k} ,
\end{aligned}$$

$$d_-^{(c)} = \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1k} + \frac{m_{u^i}}{\sqrt{2}m_w \sin \beta} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2k} \right). \quad (86)$$

## D.2 The corrections due to gluino contributions

The corrections caused by gluino can be written as

$$\phi_\alpha^{\tilde{g}} = \phi_\alpha^{w\tilde{g}} + 2\phi_\alpha^{wh\tilde{g}} + \phi_\alpha^{hh\tilde{g}} + \phi_\alpha^{sw\tilde{g}} + \phi_\alpha^{sh\tilde{g}} + \phi_\alpha^{p\tilde{g}} \quad (87)$$

with

$$\begin{aligned} \phi_1^{w\tilde{g}} = & \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \left( \frac{4}{3} F_A^{2a} + \frac{16}{3} F_A^{2b} - 4F_A^{2d} - \frac{4}{3} F_A^{2e} - \frac{16}{3} F_A^{2f} \right) (x_{jw}, 1, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}, x_{\tilde{g}w}) \\ & + \frac{16}{3} (\mathcal{Z}_{\tilde{U}^i}^{1\alpha})^2 (F_C^{2a} + F_C^{2d} - F_C^{2e}) (x_{jw}, x_{iw}, x_{iw}, 1, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\ & - \frac{16}{3} \mathcal{Z}_{\tilde{D}^3}^{1\delta} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{D}^3}^{1\delta} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} (F_D^{2e} - F_D^{2a} - F_D^{2d}) (x_{iw}, x_{jw}, 1, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{g}w}) \\ & - \frac{64}{3} \sqrt{x_{\tilde{g}w} x_{iw}} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} F_C^{1a} (x_{jw}, x_{iw}, x_{iw}, 1, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\ & + \frac{32}{3} \sqrt{x_{\tilde{g}w} x_{iw}} \mathcal{Z}_{\tilde{D}^3}^{1\delta} \mathcal{Z}_{\tilde{D}^3}^{1\delta} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} (F_D^{1b} - F_D^{1c}) (x_{iw}, x_{jw}, 1, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{g}w}) \\ & + \frac{16}{3} x_{iw} (\mathcal{Z}_{\tilde{U}^i}^{2\alpha})^2 (F_C^{1a} + F_C^{1b} - F_C^{1c}) (x_{jw}, x_{iw}, x_{iw}, 1, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\ & + \frac{56}{3} \left( \frac{x_{iw} \ln x_{iw}}{(-x_{iw} + x_{jw})^2} + \frac{x_{iw} \ln x_{iw}}{(1 - x_{iw})^2 (-x_{iw} + x_{jw})^2} - \frac{2x_{iw} \ln x_{iw}}{(1 - x_{iw})(-x_{iw} + x_{jw})^2} + \frac{\ln x_{iw}}{-x_{iw} + x_{jw}} \right. \\ & + \frac{\ln x_{iw}}{(1 - x_{iw})^2 (-x_{iw} + x_{jw})} + \frac{2x_{iw} \ln x_{iw}}{(1 - x_{iw})^3 (-x_{iw} + x_{jw})} - \frac{2x_{iw} \ln x_{iw}}{(1 - x_{iw})^2 (-x_{iw} + x_{jw})} \\ & \left. - \frac{2 \ln x_{iw}}{(1 - x_{iw})(-x_{iw} + x_{jw})} \right) - \frac{64}{3} \frac{x_{iw}^2 \ln x_{iw}}{(1 - x_{iw})^2 (-x_{iw} + x_{jw})} + \frac{16x_{iw}^2 \ln x_{iw}}{(1 - x_{iw})^2 (-x_{iw} + x_{jw})} \\ & - \frac{56}{3} \frac{x_{jw}^3 \ln x_{jw}}{(1 - x_{jw})^2 (x_{iw} - x_{jw})^2} + \frac{64}{3} \frac{x_{jw}^2 \ln x_{jw}}{(1 - x_{jw})^2 (x_{iw} - x_{jw})} + \frac{16x_{jw}^2 \ln x_{jw}}{(1 - x_{jw})^2 (x_{iw} - x_{jw})} + (i \leftrightarrow j) \end{aligned} \quad (88)$$

$$\begin{aligned} \phi_2^{w\tilde{g}} = & \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \left( \frac{2}{3} F_A^{2a} + \frac{8}{3} F_A^{2b} - 2F_A^{2d} - \frac{2}{3} F_A^{2e} - \frac{8}{3} F_A^{2f} \right) (x_{jw}, 1, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}, x_{\tilde{g}w}) \\ & + (i \leftrightarrow j), \end{aligned} \quad (89)$$

$$\phi_3^{w\tilde{g}} = -2\phi_2^{w\tilde{g}}, \quad (90)$$

$$\phi_1^{wh\tilde{g}} = -\frac{64}{3 \sin^2 \beta} (\mathcal{Z}_H^{2k})^2 \frac{x_{iw}^{\frac{5}{2}} x_{jw}^{\frac{3}{2}}}{(1 - x_{iw})(x_{H_k^- w} - x_{iw})(-x_{iw} + x_{jw})}$$

$$\begin{aligned}
& -\frac{16}{3\sin^2\beta}(x_{iw}x_{jw})^{\frac{3}{2}}(\mathcal{Z}_H^{2k})^2(\mathcal{Z}_{\tilde{U}^i}^{1\alpha})^2(F_C^{1a}+F_C^{1b}-F_C^{1c})(x_{jw},x_{iw},x_{iw},x_{H_k^-w},1,x_{\tilde{U}_{\alpha w}^i},x_{\tilde{g}w}) \\
& +\frac{32}{3\sin^2\beta}(\mathcal{Z}_H^{2k})^2\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}x_{iw}^2x_{jw}\sqrt{x_{\tilde{g}w}x_{jw}}F_C^0(x_{jw},x_{iw},x_{iw},x_{H_k^-w},1,x_{\tilde{U}_{\alpha w}^i},x_{\tilde{g}w}) \\
& +\frac{32}{3\sin^2\beta}(\mathcal{Z}_H^{2k})^2\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}x_{iw}x_{jw}\sqrt{x_{\tilde{g}w}x_{jw}}F_C^{1a}(x_{jw},x_{iw},x_{iw},x_{H_k^-w},1,x_{\tilde{U}_{\alpha w}^i},x_{\tilde{g}w}) \\
& -\frac{16}{3\sin^2\beta}(\mathcal{Z}_H^{2k})^2(\mathcal{Z}_{\tilde{U}^i}^{2\alpha})^2(x_{iw}x_{jw})^{\frac{3}{2}}(F_C^{1a}+F_C^{1b}-F_C^{1c})(x_{jw},x_{iw},x_{iw},x_{H_k^-w},1,x_{\tilde{U}_{\alpha w}^i},x_{\tilde{g}w}) \\
& -\frac{8}{3\sin^2\beta}(x_{iw}x_{jw})^{\frac{3}{2}}(\mathcal{Z}_H^{2k})^2\left(\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_{\tilde{U}^i}^{1\alpha}+\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}\right)\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_{\tilde{U}^j}^{1\beta}(F_D^{1a}-F_D^{1b} \\
& -F_D^{1c})(x_{iw},x_{jw},x_{H_k^-w},1,x_{\tilde{U}_{\alpha w}^i},x_{\tilde{D}_{\delta}^1w},x_{\tilde{g}w}) \\
& +\frac{64}{3\sin^2\beta}(x_{iw}x_{jw})^{\frac{3}{2}}(\mathcal{Z}_H^{2k})^2\frac{x_{H_k^-w}^2\ln x_{H_k^-w}}{(1-x_{H_k^-w})(-x_{H_k^-w}+x_{iw})^2(-x_{H_k^-w}+x_{jw})} \\
& +(\mathcal{Z}_H^{2k})^2(x_{iw}x_{jw})^{\frac{3}{2}}\frac{8x_{H_k^-w}\ln x_{H_k^-w}}{\sin^2\beta(1-x_{H_k^-w})(-x_{H_k^-w}+x_{iw})(-x_{H_k^-w}+x_{jw})} \\
& +\frac{4}{3}h_d\mathcal{E}^{ib}\sqrt{x_{iw}}\mathcal{Z}_H^{1j}\mathcal{Z}_{\tilde{D}^3}^{1\gamma}\mathcal{Z}_{\tilde{D}^1}^{1\delta}(F_A^{1a}-F_A^{1b}-F_A^{1c})(x_{iw},x_{H_j^-w},1,x_{\tilde{U}_{\alpha w}^i},x_{\tilde{D}_{\gamma}^3w},x_{\tilde{D}_{\delta}^1w},x_{\tilde{g}w}) \\
& -\frac{64}{3\sin^2\beta}(\mathcal{Z}_H^{2k})^2\left(-x_{H_k^-w}x_{iw}^3x_{jw}+x_{iw}^5x_{jw}+2x_{H_k^-w}x_{iw}^2x_{jw}^2-x_{iw}^3x_{jw}^2-x_{H_k^-w}x_{iw}^3x_{jw}^2\right) \\
& \frac{\sqrt{x_{iw}x_{jw}}\ln x_{iw}}{(1-x_{iw})^2(x_{H_k^-w}-x_{iw})^2(-x_{iw}+x_{jw})^2}+\frac{8}{\sin^2\beta}(\mathcal{Z}_H^{2k})^2\left(\frac{8x_{iw}^{\frac{3}{2}}x_{jw}^{\frac{7}{2}}\ln x_{jw}}{(1-x_{jw})(x_{H_k^-w}-x_{jw})(x_{iw}-x_{jw})^2}\right. \\
& \left.+\frac{x_{iw}^{\frac{5}{2}}x_{jw}^{\frac{3}{2}}\ln x_{iw}}{((1-x_{iw})(x_{H_k^-w}-x_{iw})(x_{iw}-x_{jw}))}+\frac{x_{iw}^{\frac{3}{2}}x_{jw}^{\frac{5}{2}}\ln x_{jw}}{(1-x_{jw})(x_{H_k^-w}-x_{jw})(x_{iw}-x_{jw})}\right) \\
& +(i\leftrightarrow j), \tag{91}
\end{aligned}$$

$$\begin{aligned}
\phi_2^{wh\tilde{g}} &= -\frac{4}{3}h_bh_d\left(\sqrt{x_{iw}x_{jw}}\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_{\tilde{U}^i}^{1\alpha}+\sqrt{x_{\tilde{g}w}x_{jw}}\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}\right)(\mathcal{Z}_H^{1k})^2\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_{\tilde{U}^j}^{1\beta} \\
& (F_D^{1a}-3F_D^{1b}-F_D^{1c})(x_{iw},x_{jw},x_{H_k^-w},1,x_{\tilde{U}_{\alpha w}^i},x_{\tilde{D}_{\delta}^1w},x_{\tilde{g}w}) \\
& -\frac{4x_{iw}x_{jw}}{3\sin^2\beta}\left(\sqrt{x_{\tilde{g}w}x_{jw}}\mathcal{Z}_{\tilde{D}^1}^{2\delta}\mathcal{Z}_{\tilde{U}^i}^{1\alpha}+\sqrt{x_{iw}x_{jw}}\mathcal{Z}_{\tilde{D}^1}^{2\delta}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}\right)(\mathcal{Z}_H^{2k})^2\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_{\tilde{U}^j}^{1\beta} \\
& (F_D^{1a}-3F_D^{1b}-F_D^{1c})(x_{iw},x_{jw},x_{H_k^-w},1,x_{\tilde{U}_{\alpha w}^i},x_{\tilde{D}_{\delta}^1w},x_{\tilde{g}w}) \\
& +\frac{2}{3}h_d\sqrt{x_{iw}}\mathcal{Z}_H^{1j}\mathcal{Z}_{\tilde{D}^3}^{2\gamma}\mathcal{Z}_{\tilde{D}^1}^{2\delta}\mathcal{E}^{ib}(F_A^{1a}-3F_A^{1b}-F_A^{1c})(x_{iw},x_{H_j^-w},1,x_{\tilde{U}_{\alpha w}^i},x_{\tilde{D}_{\gamma}^3w},x_{\tilde{D}_{\delta}^1w},x_{\tilde{g}w}) \\
& -\frac{4}{3}h_bh_d(\mathcal{Z}_H^{1k})^2\left(\frac{x_{H_k^-w}\sqrt{x_{iw}x_{jw}}\ln x_{H_k^-w}}{(1-x_{H_k^-w})(-x_{H_k^-w}+x_{iw})(-x_{H_k^-w}+x_{jw})}+\frac{x_{iw}\sqrt{x_{iw}x_{jw}}\ln x_{iw}}{(1-x_{iw})(x_{H_k^-w}-x_{iw})(-x_{iw}+x_{jw})}\right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{x_{jw} \sqrt{x_{iw} x_{jw}} \ln x_{jw}}{(1 - x_{jw})(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} \Big) - \frac{4}{3 \sin^2 \beta} (\mathcal{Z}_H^{2k})^2 \Big( \frac{x_{H_k^- w} x_{iw} x_{jw} \sqrt{x_{\tilde{g}w} x_{jw}} \ln x_{H_k^- w}}{(1 - x_{H_k^- w})(-x_{H_k^- w} + x_{iw})(-x_{H_k^- w} + x_{jw})} \\
& + \frac{x_{iw}^2 x_{jw} \sqrt{x_{\tilde{g}w} x_{jw}} \ln x_{iw}}{(1 - x_{iw})(x_{H_k^- w} - x_{iw})(-x_{iw} + x_{jw})} + \frac{x_{iw} x_{jw}^2 \sqrt{x_{\tilde{g}w} x_{jw}} \ln x_{jw}}{(1 - x_{jw})(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} \Big) \\
& + \frac{4}{3} h_b h_d (\mathcal{Z}_H^{1k})^2 \Big( (\mathcal{Z}_{\tilde{U}^i}^{1\alpha})^2 + (\mathcal{Z}_{\tilde{U}^i}^{1\alpha})^2 \Big) \Big( F_C^{2a} + F_C^{2d} - F_C^{2e} \Big) (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\
& - \frac{16}{3} h_b h_d \sqrt{x_{\tilde{g}w} x_{iw}} (\mathcal{Z}_H^{1k})^2 \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} F_C^{1a} (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\
& - \frac{8}{3 \sin \beta} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \Big( h_d x_{iw} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} + h_b x_{jw} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \Big) \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \\
& \Big( F_D^{2b} + F_D^{2d} - 2F_D^{2f} \Big) (x_{iw}, x_{jw}, x_{H_k^- w}, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{g}w}) \\
& + \frac{16}{3 \sin \beta} \sqrt{x_{\tilde{g}w} x_{iw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \Big( h_b x_{jw} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} + h_d x_{iw} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \Big) \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \\
& \Big( F_D^{1b} - F_D^{1c} \Big) (x_{iw}, x_{jw}, x_{H_k^- w}, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{g}w}) \\
& - \frac{8}{3 \sin \beta} \sqrt{x_{\tilde{g}w} x_{iw}} \mathcal{Z}_H^{2j} \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{E}^{ib} \Big( F_A^{1b} - F_A^{1c} \Big) (x_{iw}, x_{H_j^- w}, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\gamma^3 w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{g}w}) \\
& - 2h_b h_d (\mathcal{Z}_H^{1k})^2 \frac{x_{H_k^- w}^2 (x_{H_k^- w} + x_{iw}) \ln x_{H_k^- w}}{(-1 + x_{H_k^- w})(x_{H_k^- w} - x_{iw})^2 (x_{H_k^- w} - x_{jw})} \\
& - \frac{4}{3 \sin \beta} h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \frac{(x_{H_k^- w} + 2x_{\tilde{g}w}) x_{H_k^- w} x_{iw} \ln x_{H_k^- w}}{(1 - x_{H_k^- w})(-x_{H_k^- w} + x_{iw})(-x_{H_k^- w} + x_{jw})} \\
& + 2h_b h_d (\mathcal{Z}_H^{1k})^2 \Big( -2x_{H_k^- w} x_{iw} + x_{iw}^2 + x_{H_k^- w} x_{iw}^2 + 3x_{H_k^- w} x_{jw} - 2x_{iw} x_{jw} \\
& - 2x_{H_k^- w} x_{iw} x_{jw} + x_{iw}^2 x_{jw} \Big) \frac{x_{iw}^2 \ln x_{iw}}{(1 - x_{H_k^- w})^2 (-x_{H_k^- w} + x_{iw})^2 (-x_{H_k^- w} + x_{jw})^2} \\
& - 2h_b h_d (\mathcal{Z}_H^{1k})^2 \Big( x_{H_k^- w} x_{iw} - x_{iw}^3 - 2x_{H_k^- w} x_{jw} + x_{iw} x_{jw} \\
& + x_{H_k^- w} x_{iw} x_{jw} \Big) \frac{x_{iw}^2 \ln x_{iw}}{(x_{H_k^- w} - x_{iw})^2 (-1 + x_{iw})^2 (x_{iw} - x_{jw})^2} \\
& - 2h_b h_d (\mathcal{Z}_H^{1k})^2 \frac{(x_{iw} + x_{jw}) x_{jw}^2 \ln x_{jw}}{(x_{iw} - x_{jw})(-1 + x_{jw})(-x_{H_k^- w} + x_{jw})} \\
& - \frac{4}{3 \sin \beta} h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \frac{(-2x_{\tilde{g}w} + 3x_{jw}) x_{iw} x_{jw} \ln x_{jw}}{(x_{iw} - x_{jw})(-1 + x_{jw})(-x_{H_k^- w} + x_{jw})} \\
& + (i \leftrightarrow j) , \tag{92}
\end{aligned}$$

$$\phi_3^{wh\tilde{g}} = -2\phi_2^{wh\tilde{g}} , \tag{93}$$

$$\begin{aligned}
\phi_4^{wh\tilde{g}} = & -\frac{16}{3\sin\beta} h_d x_{iw} \sqrt{x_{\tilde{g}w} x_{iw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \left( F_D^{1b} \right. \\
& \left. - F_D^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{g}w}) \\
& + \frac{1}{3\sin\beta} h_d x_{iw} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \left( 3F_D^{2a} + 11F_D^{2b} \right. \\
& \left. + 11F_D^{2c} + 2F_D^{2d} - 14F_D^{2e} - 22F_D^{2f} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{g}w}) \\
& + \frac{8}{3\sin\beta} \sqrt{x_{\tilde{g}w} x_{iw}} \mathcal{Z}_H^{2j} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{E}^{ib} \left( F_A^{1b} - F_A^{1c} \right) (x_{iw}, x_{H_j^- w}, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\gamma^3 w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{g}w}) \\
& + \frac{1}{\sin\beta} h_d x_{iw} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \left( F_D^{2a} + F_D^{2b} + F_D^{2c} - 2F_D^{2d} \right. \\
& \left. - 2F_D^{2e} - 2F_D^{2f} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{g}w}) \\
& + (i \leftrightarrow j) ,
\end{aligned} \tag{94}$$

$$\phi_5^{wh\tilde{g}} = \frac{1}{4} \phi_4^{wh\tilde{g}} , \tag{95}$$

$$\begin{aligned}
\phi_6^{wh\tilde{g}} = & -\frac{8}{3} h_b h_d \sqrt{x_{\tilde{g}w} x_{jw}} (\mathcal{Z}_H^{1k})^2 \left( \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} + \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \right) \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \left( F_D^{1a} - 3F_D^{1b} \right. \\
& \left. - F_D^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{g}w}) \\
& + (i \leftrightarrow j) ,
\end{aligned} \tag{96}$$

$$\begin{aligned}
\phi_7^{wh\tilde{g}} = & \frac{1}{\sin\beta} h_b x_{jw} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \left( 2F_D^{2a} + \frac{14}{3} F_D^{2b} + \frac{14}{3} F_D^{2c} - \frac{4}{3} F_D^{2d} \right. \\
& \left. - \frac{20}{3} F_D^{2e} - \frac{28}{3} F_D^{2f} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{g}w}) \\
& - \frac{16}{3\sin\beta} h_b \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \sqrt{x_{\tilde{g}w} x_{iw}} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \left( F_D^{1b} - F_D^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, 1, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{g}w}) \\
& - \frac{35h_b \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k}}{3\sin\beta} \frac{x_{\tilde{g}w}^2 x_{H_k^- w} x_{jw} \ln x_{H_k^- w}}{(x_{\tilde{D}_\delta^1 w} - x_{\tilde{g}w})(1 - x_{H_k^- w})(-x_{H_k^- w} + x_{iw})(-x_{H_k^- w} + x_{jw})} \\
& - \frac{h_b \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k}}{3\sin\beta} \frac{19x_{\tilde{g}w}^2 x_{iw} x_{jw} \ln x_{iw} - 3(x_{\tilde{D}_\delta^1 w} + x_{\tilde{g}w}) x_{iw}^2 x_{jw} \ln x_{iw}}{(x_{\tilde{D}_\delta^1 w} - x_{\tilde{g}w})(1 - x_{iw})(x_{H_k^- w} - x_{iw})(-x_{iw} + x_{jw})} \\
& - \frac{h_b \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k}}{3\sin\beta} \frac{19x_{\tilde{g}w}^2 x_{jw}^2 \ln x_{jw}}{3\sin\beta (x_{\tilde{D}_\delta^1 w} - x_{\tilde{g}w})(1 - x_{jw})(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} \Big) \\
& + \frac{h_b \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k}}{2\sin\beta} \frac{32x_{H_k^- w}^2 x_{jw} \ln x_{H_k^- w} + 21x_{H_k^- w} x_{\tilde{g}w} x_{jw} \ln x_{H_k^- w}}{(1 - x_{H_k^- w})(-x_{H_k^- w} + x_{iw})(-x_{H_k^- w} + x_{jw})}
\end{aligned}$$



$$\begin{aligned}
& + \frac{h_b \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k}}{2 \sin \beta} \frac{(32x_{iw} + 25x_{\bar{g}w})x_{iw}x_{jw} \ln x_{iw}}{(1 - x_{iw})(x_{H_k^- w} - x_{iw})(x_{jw} - x_{iw})} \\
& + \frac{h_b \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k}}{6 \sin \beta} \frac{(166x_{jw} + 39x_{\bar{g}w})x_{jw}^2 \ln x_{jw}}{(1 - x_{jw})(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})} \\
& - \frac{20h_b \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k}}{\sin \beta} \frac{x_{H_k^- w} x_{jw} \ln x_{H_k^- w}}{\sin \beta (-x_{H_k^- w} + x_{iw})(-x_{H_k^- w} + x_{jw})} \\
& + \frac{35h_b \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k}}{\sin \beta} \frac{x_{iw} x_{jw} \ln x_{iw}}{\sin \beta (-x_{H_k^- w} + x_{iw})(-x_{iw} + x_{jw})} \\
& + \frac{35h_b \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k}}{\sin \beta} \frac{x_{jw}^2 \ln x_{iw}}{\sin \beta (-x_{H_k^- w} + x_{jw})(-x_{iw} + x_{jw})} \\
& + (i \leftrightarrow j) ,
\end{aligned} \tag{97}$$

$$\phi_8^{wh\bar{g}} = \frac{1}{4} \phi_7^{wh\bar{g}} , \tag{98}$$

$$\begin{aligned}
\phi_1^{hh\bar{g}} = & \frac{8}{3 \sin^3 \beta} x_{iw}^{\frac{5}{2}} x_{jw} \mathcal{Z}_H^{2k} (\mathcal{Z}_H^{2l})^2 \mathcal{Z}_{\bar{D}1}^{1\gamma} \mathcal{Z}_{\bar{U}i}^{1\alpha} \mathcal{E}^{id} (F_D^{1a} + F_D^{1b} - F_D^{1c}) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{H_l^- w}, x_{\bar{U}_\alpha^i w}, x_{\bar{D}_\gamma^1 w}, x_{\bar{g}w}) \\
& + \frac{4}{3 \sin^4 \beta} x_{iw}^3 x_{jw}^2 (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 (\mathcal{Z}_{\bar{U}i}^{1\alpha})^2 (F_C^{1a} + F_C^{1b} - F_C^{1c}) (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\bar{U}_\alpha^i w}, x_{\bar{g}w}) \\
& - \frac{16}{3 \sin^3 \beta} \sqrt{x_{\bar{g}w}} x_{iw}^2 x_{jw} \mathcal{Z}_H^{2k} (\mathcal{Z}_H^{2l})^2 \mathcal{Z}_{\bar{D}1}^{1\gamma} \mathcal{E}^{id} \mathcal{Z}_{\bar{U}i}^{2\alpha} F_D^{1a} (x_{iw}, x_{jw}, x_{H_k^- w}, x_{H_l^- w}, x_{\bar{U}_\alpha^i w}, x_{\bar{D}_\gamma^1 w}, x_{\bar{g}w}) \\
& - \frac{16}{3 \sin^4 \beta} x_{iw}^2 \sqrt{x_{\bar{g}w}} x_{iw} x_{jw}^2 (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \mathcal{Z}_{\bar{U}i}^{1\alpha} \mathcal{Z}_{\bar{U}i}^{2\alpha} F_C^{1a} (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\bar{U}_\alpha^i w}, x_{\bar{g}w}) \\
& + \frac{4}{3 \sin^4 \beta} x_{iw}^2 x_{jw}^2 (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 (\mathcal{Z}_{\bar{U}i}^{2\alpha})^2 (F_C^{2a} + F_D^{2d} - F_D^{2e}) (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\bar{U}_\alpha^i w}, x_{\bar{g}w}) \\
& + \frac{4}{3} h_b h_d \mathcal{Z}_H^{1i} \mathcal{Z}_H^{1k} \mathcal{Z}_{\bar{D}3}^{1\delta} \mathcal{Z}_{\bar{D}1}^{1\gamma} \mathcal{E}^{ib} \mathcal{E}^{jd} (F_A^{1a} + F_A^{1b} - F_A^{1c}) (x_{jw}, x_{H_l^- w}, x_{H_k^- w}, x_{\bar{U}_\alpha^i w}, x_{\bar{D}_\delta^3 w}, x_{\bar{D}_\gamma^1 w}, x_{\bar{g}w}) \\
& + \frac{8}{\sin^3 \beta} \mathcal{Z}_H^{2k} (\mathcal{Z}_H^{2l})^2 \mathcal{Z}_{\bar{D}1}^{1\gamma} \mathcal{Z}_{\bar{U}i}^{1\alpha} \mathcal{E}^{id} \frac{x_{H_k^- w} x_{iw}^{\frac{5}{2}} x_{jw} \ln x_{H_k^- w}}{(-x_{H_k^- w} + x_{H_l^- w})(-x_{H_k^- w} + x_{iw})(-x_{H_k^- w} + x_{jw})} \\
& - \frac{2}{\sin^4 \beta} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \frac{x_{H_k^- w}^2 x_{iw}^3 x_{jw}^2 \ln x_{H_k^- w}}{(-x_{H_k^- w} + x_{H_l^- w})(-x_{H_k^- w} + x_{iw})^2 (-x_{H_k^- w} + x_{jw})} \\
& + \frac{2}{\sin^4 \beta} \left( \frac{(\mathcal{Z}_H^{2k})^2 x_{H_k^- w}^2 x_{iw}^2 x_{jw}^2 \ln x_{H_k^- w}}{(-x_{H_k^- w} + x_{iw})^2 (-x_{H_k^- w} + x_{jw})} - \frac{(\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 x_{H_k^- w}^2 x_{iw}^2 x_{jw}^2 \ln x_{H_k^- w}}{(-x_{H_k^- w} + x_{H_l^- w})(-x_{H_k^- w} + x_{iw})^2 (-x_{H_k^- w} + x_{jw})} \right) \\
& + \frac{8}{\sin^3 \beta} \mathcal{Z}_H^{2k} (\mathcal{Z}_H^{2l})^2 \mathcal{Z}_{\bar{D}1}^{1\gamma} \mathcal{E}^{id} \frac{x_{H_l^- w} x_{iw}^{\frac{5}{2}} x_{jw} \ln x_{H_l^- w}}{(x_{H_k^- w} - x_{H_l^- w})(-x_{H_l^- w} + x_{iw})(-x_{H_l^- w} + x_{jw})}
\end{aligned}$$

$$\begin{aligned}
& -\frac{2}{\sin^4 \beta} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \frac{x_{H_l^- w}^2 x_{iw}^3 x_{jw}^2 \ln x_{H_l^- w}}{(x_{H_k^- w} - x_{H_l^- w})(-x_{H_l^- w} + x_{iw})^2 (-x_{H_l^- w} + x_{jw})} \\
& -\frac{2}{\sin^4 \beta} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \frac{x_{H_l^- w}^3 x_{iw}^2 x_{jw}^2 \ln x_{H_l^- w}}{(x_{H_k^- w} - x_{H_l^- w})(-x_{H_l^- w} + x_{iw})^2 (-x_{H_l^- w} + x_{jw})} \\
& +\frac{8}{\sin^3 \beta} \mathcal{Z}_H^{2k} (\mathcal{Z}_H^{2l})^2 \mathcal{Z}_{\tilde{D}1}^{1\gamma} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{E}^{id} \frac{x_{iw}^{\frac{7}{2}} x_{jw} \ln x_{iw}}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(-x_{iw} + x_{jw})} \\
& +\frac{2}{\sin^4 \beta} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \left( \frac{x_{iw}^5 x_{jw}^2 \ln x_{iw}}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(-x_{iw} + x_{jw})^2} \right. \\
& +\frac{x_{iw}^4 x_{jw}^2 \ln x_{iw}}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(-x_{iw} + x_{jw})} + \frac{x_{iw}^5 x_{jw}^2 \ln x_{iw}}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})^2 (-x_{iw} + x_{jw})} \\
& \left. +\frac{x_{H_l^- w} x_{iw}^4 x_{jw}^2 \ln x_{iw}}{(x_{H_k^- w} - x_{iw})^2 (x_{H_l^- w} - x_{iw})(-x_{iw} + x_{jw})} \right) \\
& +\frac{2}{\sin^4 \beta} \left( \frac{(\mathcal{Z}_H^{2k})^2 x_{iw}^4 x_{jw}^2 \ln x_{iw}}{(x_{H_k^- w} - x_{iw})(-x_{iw} + x_{jw})^2} + \frac{(\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 x_{iw}^4 x_{jw}^2 \ln x_{iw}}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(-x_{iw} + x_{jw})^2} \right. \\
& +\frac{(\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 x_{iw}^3 x_{jw}^2 \ln x_{iw}}{(x_{H_k^- w} - x_{iw})(-x_{iw} + x_{jw})} + \frac{(\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 x_{iw}^3 x_{jw}^2 \ln x_{iw}}{(x_{H_k^- w} - x_{iw})(x_{H_l^- w} - x_{iw})(-x_{iw} + x_{jw})} \Big) \\
& +\frac{8}{\sin^3 \beta} \mathcal{Z}_H^{2k} (\mathcal{Z}_H^{2l})^2 \mathcal{Z}_{\tilde{D}1}^{1\gamma} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{E}^{id} \frac{x_{iw}^{\frac{5}{2}} x_{jw}^2 \ln x_{jw}}{(x_{H_k^- w} - x_{jw})(x_{H_l^- w} - x_{jw})(x_{iw} - x_{jw})} \\
& -\frac{2}{\sin^4 \beta} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \frac{(1 + x_{iw}) x_{iw}^2 x_{jw}^4 \ln x_{jw}}{(x_{H_k^- w} - x_{jw})(x_{H_l^- w} - x_{jw})(x_{iw} - x_{jw})^2} \\
& +\frac{2}{\sin^4 \beta} (\mathcal{Z}_H^{2k})^2 (\mathcal{Z}_H^{2l})^2 \frac{x_{iw}^2 x_{jw}^4 \ln x_{jw}}{(x_{H_k^- w} - x_{jw})(x_{iw} - x_{jw})^2} + (i \leftrightarrow j) , \tag{99}
\end{aligned}$$

$$\begin{aligned}
\phi_2^{hh\tilde{g}} &= \frac{4}{3 \sin \beta} h_b h_d x_{iw}^{\frac{3}{2}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{1\gamma} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{E}^{id} \left( F_D^{1a} + F_D^{1b} - F_D^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\gamma^1 w}, x_{\tilde{g}w}) \\
& -\frac{8}{3 \sin^2 \beta} h_b \sqrt{x_{\tilde{g}w}} x_{iw} x_{jw} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{E}^{id} \left( F_D^{1a} + F_D^{1b} \right. \\
& \left. - F_D^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\gamma^1 w}, x_{\tilde{g}w}) \\
& +\frac{2}{3 \sin^2 \beta} h_b h_d x_{iw}^3 \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} (\mathcal{Z}_{\tilde{U}^i}^{1\alpha})^2 \left( F_C^{1a} + F_C^{1b} - F_C^{1c} \right) (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\
& +\frac{2}{3 \sin^2 \beta} h_b h_d x_{jw}^2 \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} (\mathcal{Z}_{\tilde{U}^i}^{1\alpha})^2 \left( F_C^{2a} + F_C^{2d} - F_C^{2e} \right) (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w})
\end{aligned}$$

$$\begin{aligned}
& -\frac{8}{3\sin^2\beta}h_bh_d\sqrt{x_{\tilde{g}w}}x_{iw}\mathcal{Z}_H^{1k}\mathcal{Z}_H^{1l}\mathcal{Z}_H^{2l}\mathcal{Z}_{\tilde{D}1}^{1\gamma}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}\mathcal{E}^{id}\left(F_D^{1a}+F_D^{1b}\right. \\
& \left.-F_D^{1c}\right)(x_{iw},x_{jw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{D}_\gamma^1w},x_{\tilde{g}w}) \\
& +\frac{4}{3\sin^2\beta}h_bx_{iw}^{\frac{3}{2}}x_{jw}\mathcal{Z}_H^{1l}\mathcal{Z}_H^{2k}\mathcal{Z}_H^{2l}\mathcal{Z}_{\tilde{D}1}^{2\gamma}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}\mathcal{E}^{id}\left(F_D^{1a}+F_D^{1b}-F_D^{1c}\right)(x_{iw},x_{jw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{D}_\gamma^1w},x_{\tilde{g}w}) \\
& -\frac{8}{3\sin^2\beta}h_bh_d(x_{iw}^2+x_{jw}^2)\sqrt{x_{\tilde{g}w}}x_{iw}\mathcal{Z}_H^{1k}\mathcal{Z}_H^{1l}\mathcal{Z}_H^{2k}\mathcal{Z}_H^{2l}\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}F_C^{1a}(x_{jw},x_{iw},x_{iw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{g}w}) \\
& +\frac{2}{3\sin^2\beta}h_bh_dx_{iw}x_{jw}^2\mathcal{Z}_H^{1k}\mathcal{Z}_H^{1l}\mathcal{Z}_H^{2k}\mathcal{Z}_H^{2l}(\mathcal{Z}_{\tilde{U}^i}^{2\alpha})^2\left(F_C^{1a}+F_C^{1b}-F_C^{1c}\right)(x_{jw},x_{iw},x_{iw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{g}w}) \\
& +\frac{2}{3\sin^2\beta}h_bh_dx_{iw}^2\mathcal{Z}_H^{1k}\mathcal{Z}_H^{1l}\mathcal{Z}_H^{2k}\mathcal{Z}_H^{2l}(\mathcal{Z}_{\tilde{U}^i}^{2\alpha})^2\left(F_C^{2a}+F_C^{2d}-F_C^{2e}\right)(x_{jw},x_{iw},x_{iw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{g}w}) \\
& +\frac{2}{3\sin^2\beta}x_{jw}^2\mathcal{Z}_H^{2i}\mathcal{Z}_H^{2k}\mathcal{Z}_{\tilde{D}3}^{1\delta}\mathcal{Z}_{\tilde{D}1}^{1\gamma}\mathcal{E}^{ib}\mathcal{E}^{jd}\left(F_A^{1a}+F_A^{1b}-F_A^{1c}\right)(x_{jw},x_{H_i^-w},x_{H_k^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{D}_\gamma^1w},x_{\tilde{D}_\delta^3w},x_{\tilde{g}w}) \\
& -\frac{8}{3\sin^2\beta}h_dx_{iw}x_{jw}^{\frac{3}{2}}\mathcal{Z}_H^{1l}\mathcal{Z}_H^{2k}\mathcal{Z}_H^{2l}\mathcal{Z}_{\tilde{D}1}^{1\gamma}\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{E}^{id}\left(F_D^{1a}+F_D^{1b}-F_D^{1c}\right)(x_{iw},x_{jw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{D}_\gamma^1w},x_{\tilde{g}w}) \\
& +\frac{16}{3\sin^2\beta}h_bx_{iw}^2\sqrt{x_{\tilde{g}w}}x_{iw}x_{jw}\mathcal{Z}_H^{1k}(\mathcal{Z}_H^{2l})^2\mathcal{Z}_{\tilde{D}1}^{2\gamma}\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{E}^{id}F_D^0(x_{iw},x_{jw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{D}_\gamma^1w},x_{\tilde{g}w}) \\
& -\frac{4}{3\sin^2\beta}h_bh_d(x_{iw}x_{jw})^{\frac{3}{2}}((\mathcal{Z}_H^{1l})^2(\mathcal{Z}_H^{2k})^2+(\mathcal{Z}_H^{2l})^2(\mathcal{Z}_H^{1k})^2)(\mathcal{Z}_{\tilde{U}^i}^{1\alpha})^2\left(F_C^{1a}+F_C^{1b}\right. \\
& \left.-F_C^{1c}\right)(x_{jw},x_{iw},x_{iw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{g}w}) \\
& +\frac{16}{3\sin\beta}h_bh_dx_{jw}\sqrt{x_{\tilde{g}w}}x_{iw}x_{jw}(\mathcal{Z}_H^{1l})^2\mathcal{Z}_H^{2k}\mathcal{Z}_{\tilde{D}1}^{1\gamma}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}\mathcal{E}^{id}F_D^0(x_{iw},x_{jw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{D}_\gamma^1w},x_{\tilde{g}w}) \\
& -\frac{8}{3\sin\beta}h_b^2x_{iw}^{\frac{3}{2}}\mathcal{Z}_H^{1k}\mathcal{Z}_H^{1l}\mathcal{Z}_H^{2l}\mathcal{Z}_{\tilde{D}1}^{2\gamma}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}\mathcal{E}^{id}\left(F_D^{1a}+F_D^{1b}-F_D^{1c}\right)(x_{iw},x_{jw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{D}_\gamma^1w},x_{\tilde{g}w}) \\
& +\frac{8}{3\sin^2\beta}h_bh_dx_{iw}^{\frac{5}{2}}x_{jw}^{\frac{3}{2}}((\mathcal{Z}_H^{1l})^2(\mathcal{Z}_H^{2k})^2+(\mathcal{Z}_H^{2l})^2(\mathcal{Z}_H^{1k})^2)\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{Z}_{\tilde{U}^i}^{2\alpha} \\
& F_C^0(x_{jw},x_{iw},x_{iw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{g}w}) \\
& +\frac{8}{3\sin^2\beta}h_bh_dx_{iw}x_{jw}\sqrt{x_{\tilde{g}w}}x_{jw}((\mathcal{Z}_H^{1l})^2(\mathcal{Z}_H^{2k})^2+(\mathcal{Z}_H^{2l})^2(\mathcal{Z}_H^{1k})^2)\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{Z}_{\tilde{U}^i}^{2\alpha} \\
& F_C^{1a}(x_{jw},x_{iw},x_{iw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{g}w}) \\
& -\frac{4}{3\sin^2\beta}h_bh_d(x_{iw}x_{jw})^{\frac{3}{2}}((\mathcal{Z}_H^{1l})^2(\mathcal{Z}_H^{2k})^2+(\mathcal{Z}_H^{2l})^2(\mathcal{Z}_H^{1k})^2)(\mathcal{Z}_{\tilde{U}^i}^{2\alpha})^2\left(F_C^{1a}+F_C^{1b}\right. \\
& \left.-F_C^{1c}\right)(x_{jw},x_{iw},x_{iw},x_{H_k^-w},x_{H_l^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{g}w}) \\
& +\frac{8}{3\sin\beta}h_bx_{jw}\sqrt{x_{\tilde{g}w}}x_{jw}(\mathcal{Z}_H^{1i}\mathcal{Z}_H^{2k}+\mathcal{Z}_H^{2i}\mathcal{Z}_H^{1k})\mathcal{Z}_{\tilde{D}3}^{2\delta}\mathcal{Z}_{\tilde{D}1}^{1\gamma}\mathcal{E}^{ib}\mathcal{E}^{jd} \\
& F_A^0(x_{jw},x_{H_i^-w},x_{H_k^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{D}_m^1w},x_{\tilde{D}_\gamma^1w},x_{\tilde{g}w})
\end{aligned}$$

$$\begin{aligned}
& -\frac{4}{3\sin^2\beta}h_b h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{1\gamma} \mathcal{E}^{id} \frac{x_{H_k^-w} x_{iw}^2 (\mathcal{Z}_{\tilde{U}i}^{1\alpha} \sqrt{x_{iw}} - 2\mathcal{Z}_{\tilde{U}i}^{2\alpha} \sqrt{x_{\tilde{g}w}}) \ln x_{H_k^-w}}{(-x_{H_k^-w} + x_{H_l^-w})(-x_{H_k^-w} + x_{iw})(-x_{H_k^-w} + x_{jw})} \\
& +\frac{4}{3\sin^2\beta}h_b \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{E}^{id} \frac{(2\mathcal{Z}_{\tilde{U}i}^{1\alpha} \sqrt{x_{\tilde{g}w}} - \mathcal{Z}_{\tilde{U}i}^{2\alpha} \sqrt{x_{iw}}) x_{H_k^-w} x_{iw} x_{jw} \ln x_{H_k^-w}}{(-x_{H_k^-w} + x_{H_l^-w})(-x_{H_k^-w} + x_{iw})(-x_{H_k^-w} + x_{jw})} \\
& -\frac{1}{3\sin^2\beta}h_b h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \frac{(x_{iw}^2 + x_{jw}^2)(x_{H_k^-w}^2 x_{iw} + x_{H_k^-w}^3) \ln x_{H_k^-w}}{(-x_{H_k^-w} + x_{H_l^-w})(-x_{H_k^-w} + x_{iw})^2(-x_{H_k^-w} + x_{jw})} \\
& +\frac{1}{3\sin^2\beta}h_b h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \frac{x_{H_k^-w}^2 x_{iw}^2 \ln(x_{H_k^-w})}{(-x_{H_k^-w} + x_{iw})^2(-x_{H_k^-w} + x_{jw})} \\
& -\frac{4}{3\sin\beta}h_b h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{1\gamma} \mathcal{E}^{id} \frac{(\mathcal{Z}_{\tilde{U}i}^{1\alpha} \sqrt{x_{iw}} - 2\mathcal{Z}_{\tilde{U}i}^{2\alpha} \sqrt{x_{\tilde{g}w}}) x_{H_l^-w} x_{iw} \ln x_{H_l^-w}}{(x_{H_k^-w} - x_{H_l^-w})(-x_{H_l^-w} + x_{iw})(-x_{H_l^-w} + x_{jw})} \\
& +\frac{4}{3\sin^2\beta}h_b \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{E}^{id} \frac{(2\mathcal{Z}_{\tilde{U}i}^{1\alpha} \sqrt{x_{\tilde{g}w}} - \mathcal{Z}_{\tilde{U}i}^{2\alpha} \sqrt{x_{iw}}) x_{H_l^-w} x_{iw} x_{jw} \ln x_{H_l^-w}}{(x_{H_k^-w} - x_{H_l^-w})(-x_{H_l^-w} + x_{iw})(-x_{H_l^-w} + x_{jw})} \\
& -\frac{1}{3\sin^2\beta}h_b h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \frac{(x_{iw}^2 + x_{jw}^2)(x_{H_l^-w}^2 x_{iw} + x_{H_l^-w}^3) \ln x_{H_l^-w}}{(x_{H_k^-w} - x_{H_l^-w})(-x_{H_l^-w} + x_{iw})^2(-x_{H_l^-w} + x_{jw})} \\
& -\frac{4}{3\sin\beta}h_b h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{1\gamma} \mathcal{E}^{id} \frac{(\mathcal{Z}_{\tilde{U}i}^{1\alpha} \sqrt{x_{iw}} - 2\mathcal{Z}_{\tilde{U}i}^{2\alpha} \sqrt{x_{\tilde{g}w}}) x_{iw}^2 \ln x_{iw}}{(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})(-x_{iw} + x_{jw})} \\
& +\frac{8}{3\sin^2\beta}h_b \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{Z}_{\tilde{U}i}^{1\alpha} \mathcal{E}^{id} \frac{\sqrt{x_{\tilde{g}w}} x_{iw}^2 x_{jw} \ln x_{iw}}{(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})(-x_{iw} + x_{jw})} \\
& -\frac{1}{3\sin^2\beta}h_b h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \left( 3x_{H_k^-w} x_{H_l^-w} x_{iw}^5 - x_{jw}^5 (x_{iw}^2 + x_{jw}^2) - 5x_{H_k^-w} x_{H_l^-w} x_{iw}^4 x_{jw} \right. \\
& +3(x_{H_k^-w} + x_{H_l^-w}) x_{iw}^5 x_{jw} + 3x_{H_k^-w} x_{H_l^-w} x_{iw}^3 x_{jw}^2 - (x_{H_k^-w} + x_{H_l^-w}) x_{iw}^4 (x_{iw}^2 + x_{jw}^2) \\
& \left. -5x_{H_k^-w} x_{H_l^-w} x_{iw}^2 x_{jw}^3 + 3x_{H_k^-w} x_{iw}^3 x_{jw}^3 + 3x_{H_l^-w} x_{iw}^3 x_{jw}^3 - x_{iw}^4 x_{jw} (x_{iw}^2 + x_{jw}^2) \right) \\
& \frac{\ln x_{iw}}{(x_{H_k^-w} - x_{iw})^2 (x_{H_l^-w} - x_{iw})^2 (-x_{iw} + x_{jw})^2} \\
& -\frac{4}{3\sin^2\beta}h_b \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{E}^{id} \frac{(\mathcal{Z}_{\tilde{U}i}^{2\alpha} \sqrt{x_{iw}} - 2\mathcal{Z}_{\tilde{U}i}^{1\alpha} \sqrt{x_{\tilde{g}w}}) x_{iw}^2 x_{jw} \ln x_{iw}}{(x_{H_k^-w} - x_{iw})(x_{H_l^-w} - x_{iw})(-x_{iw} + x_{jw})} \\
& -\frac{4}{3\sin\beta}h_b h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{1\gamma} \mathcal{E}^{id} \frac{(\mathcal{Z}_{\tilde{U}i}^{1\alpha} \sqrt{x_{iw}} - 2\mathcal{Z}_{\tilde{U}i}^{2\alpha} \sqrt{x_{\tilde{g}w}}) x_{iw} x_{jw} \ln x_{jw}}{(x_{H_k^-w} - x_{jw})(x_{H_l^-w} - x_{jw})(x_{iw} - x_{jw})} \\
& -\frac{4}{3\sin^2\beta}h_b \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{Z}_{\tilde{U}i}^{2\alpha} \mathcal{E}^{id} \frac{x_{iw}^{\frac{3}{2}} x_{jw}^2 \ln x_{jw}}{(x_{H_k^-w} - x_{jw})(x_{H_l^-w} - x_{jw})(x_{iw} - x_{jw})}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{3 \sin^2 \beta} h_b h_d \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \frac{(x_{iw}^3 + x_{iw}^2 x_{jw} + x_{iw} x_{jw}^2 + x_{jw}^3) x_{jw}^2 \ln x_{jw}}{(x_{H_k^- w} - x_{jw})(x_{H_l^- w} - x_{jw})(x_{iw} - x_{jw})^2} \\
& + (i \leftrightarrow j) ,
\end{aligned} \tag{100}$$

$$\phi_3^{hh\tilde{g}} = -2\phi_2^{hh\tilde{g}} , \tag{101}$$

$$\begin{aligned}
\phi_4^{hh\tilde{g}} = & -\frac{16}{3 \sin^2 \beta} h_d (x_{iw} x_{jw})^{\frac{3}{2}} \sqrt{x_{\tilde{g}w}} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{Z}_{\tilde{U}i}^{1\alpha} \mathcal{E}^{id} F_D^0(x_{iw}, x_{jw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\gamma^1 w}, x_{\tilde{g}w}) \\
& + \frac{4}{3 \sin^2 \beta} h_d^2 (x_{iw} x_{jw})^{\frac{3}{2}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \left( (\mathcal{Z}_{\tilde{U}i}^{1\alpha})^2 + (\mathcal{Z}_{\tilde{U}i}^{2\alpha})^2 \right) \left( F_C^{1a} + F_C^{1b} \right. \\
& \left. - F_C^{1c} \right) (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\
& - \frac{8}{3 \sin^2 \beta} h_d^2 x_{iw} x_{jw} \sqrt{x_{\tilde{g}w} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{U}i}^{1\alpha} \mathcal{Z}_{\tilde{U}i}^{2\alpha} F_C^{1a} (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\
& + \frac{8}{3 \sin^2 \beta} h_d x_{iw} x_{jw}^{\frac{3}{2}} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{Z}_{\tilde{U}i}^{2\alpha} \mathcal{E}^{id} \left( F_D^{1a} + F_D^{1b} - F_D^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\gamma^1 w}, x_{\tilde{g}w}) \\
& - \frac{8}{3 \sin^2 \beta} h_d^2 x_{iw}^{\frac{5}{2}} x_{jw}^{\frac{3}{2}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{U}i}^{1\alpha} \mathcal{Z}_{\tilde{U}i}^{2\alpha} F_C^0 (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\
& - \frac{8}{3 \sin \beta} h_b x_{jw}^{\frac{3}{2}} \sqrt{x_{\tilde{g}w}} \mathcal{Z}_H^{1i} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}3}^{1\delta} \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{E}^{ib} \mathcal{E}^{jd} F_A^0 (x_{jw}, x_{H_i^- w}, x_{H_k^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_m^1 w}, x_{\tilde{D}_\gamma^1 w}, x_{\tilde{g}w}) \\
& + (i \leftrightarrow j) ,
\end{aligned} \tag{102}$$

$$\phi_5^{hh\tilde{g}} = \frac{1}{4} \phi_4^{hh\tilde{g}} , \tag{103}$$

$$\begin{aligned}
\phi_6^{hh\tilde{g}} = & -\frac{16}{3} h_b^2 h_d \sqrt{x_{\tilde{g}w}} \mathcal{Z}_H^{1k} (\mathcal{Z}_H^{1l})^2 \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{Z}_{\tilde{U}i}^{1\alpha} \mathcal{E}^{id} F_D^{1a} (x_{iw}, x_{jw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\gamma^1 w}, x_{\tilde{g}w}) \\
& - \frac{16}{3} h_b^2 h_d^2 \sqrt{x_{\tilde{g}w} x_{iw}} (\mathcal{Z}_H^{1k})^2 (\mathcal{Z}_H^{1l})^2 \mathcal{Z}_{\tilde{U}i}^{1\alpha} \mathcal{Z}_{\tilde{U}i}^{2\alpha} F_C^{1a} (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\
& + \frac{4}{3} h_b^2 h_d^2 (\mathcal{Z}_H^{1k})^2 (\mathcal{Z}_H^{1l})^2 (\mathcal{Z}_{\tilde{U}i}^{1\alpha})^2 \left( F_C^{2a} + F_C^{2d} - F_C^{2e} \right) (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\
& + \frac{8}{3} h_b^2 h_d \sqrt{x_{iw}} \mathcal{Z}_H^{1k} (\mathcal{Z}_H^{1l})^2 \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{Z}_{\tilde{U}i}^{2\alpha} \mathcal{E}^{id} \left( F_D^{1a} + F_D^{1b} - F_D^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\gamma^1 w}, x_{\tilde{g}w}) \\
& + \frac{4}{3} h_b^2 h_d^2 x_{iw} (\mathcal{Z}_H^{1k})^2 (\mathcal{Z}_H^{1l})^2 (\mathcal{Z}_{\tilde{U}i}^{2\alpha})^2 \left( F_C^{1a} + F_C^{1b} - F_C^{1c} \right) (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\
& + \frac{4}{3 \sin^2 \beta} x_{jw}^2 \mathcal{Z}_H^{2i} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}3}^{2\delta} \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{E}^{ib} \mathcal{E}^{jd} \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{jw}, x_{H_i^- w}, x_{H_k^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_m^1 w}, x_{\tilde{D}_\gamma^1 w}, x_{\tilde{g}w}) \\
& - 2h_b^2 h_d^2 (\mathcal{Z}_H^{1k})^2 (\mathcal{Z}_H^{1l})^2 \frac{x_{H_k^- w}^3 \ln x_{H_k^- w}}{(-x_{H_k^- w} + x_{H_l^- w})(-x_{H_k^- w} + x_{iw})^2 (-x_{H_k^- w} + x_{jw})}
\end{aligned}$$

$$\begin{aligned}
& -2h_b^2 h_d^2 (\mathcal{Z}_H^{1k})^2 (\mathcal{Z}_H^{1l})^2 \frac{x_{H_l^- w}^3 \ln x_{H_l^- w}}{(-x_{H_l^- w} + x_{H_k^- w})(-x_{H_l^- w} + x_{iw})^2 (-x_{H_l^- w} + x_{jw})} \\
& -2h_b^2 h_d^2 (\mathcal{Z}_H^{1k})^2 (\mathcal{Z}_H^{1l})^2 \frac{x_{H_k^- w}^2 x_{iw} \ln x_{H_k^- w}}{(-x_{H_k^- w} + x_{H_l^- w})(-x_{H_k^- w} + x_{iw})^2 (-x_{H_k^- w} + x_{jw})} \\
& -2h_b^2 h_d^2 (\mathcal{Z}_H^{1k})^2 (\mathcal{Z}_H^{1l})^2 \frac{x_{H_l^- w}^2 x_{iw} \ln x_{H_l^- w}}{(-x_{H_l^- w} + x_{H_k^- w})(-x_{H_l^- w} + x_{iw})^2 (-x_{H_l^- w} + x_{jw})} \\
& +2h_b^2 h_d^2 (\mathcal{Z}_H^{1k})^2 (\mathcal{Z}_H^{1l})^2 \left( -3x_{H_k^- w} x_{H_l^- w} x_{iw}^3 + x_{H_k^- w} x_{iw}^4 + x_{H_l^- w} x_{iw}^4 \right. \\
& \left. + x_{iw}^5 + 5x_{H_k^- w} x_{H_l^- w} x_{iw}^2 x_{jw} - 3x_{H_k^- w} x_{iw}^3 x_{jw} - 3x_{H_l^- w} x_{iw}^3 x_{jw} + x_{iw}^4 x_{jw} \right) \\
& \frac{\ln x_{H_k^- w}}{(-x_{H_l^- w} + x_{H_k^- w})^2 (-x_{H_l^- w} + x_{iw})^2 (-x_{H_l^- w} + x_{jw})^2} \\
& -2h_b^2 h_d^2 (\mathcal{Z}_H^{1k})^2 (\mathcal{Z}_H^{1l})^2 \frac{(x_{iw} x_{jw}^2 + x_{jw}^3) \ln x_{jw}}{(x_{H_k^- w} - x_{jw})(x_{H_l^- w} - x_{jw})(x_{iw} - x_{jw})^2} \\
& +8h_b^2 h_d \mathcal{Z}_H^{1k} (\mathcal{Z}_H^{1l})^2 \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{Z}_{\tilde{U}i}^{2\alpha} \mathcal{E}^{id} \frac{x_{H_l^- w} \sqrt{x_{iw}} \ln x_{H_l^- w}}{(x_{H_k^- w} - x_{H_l^- w})(-x_{H_l^- w} + x_{iw})(-x_{H_l^- w} + x_{jw})} \\
& +8h_b^2 h_d \mathcal{Z}_H^{1k} (\mathcal{Z}_H^{1l})^2 \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{Z}_{\tilde{U}i}^{2\alpha} \mathcal{E}^{id} \frac{x_{H_k^- w} \sqrt{x_{iw}} \ln x_{H_k^- w}}{(x_{H_l^- w} - x_{H_k^- w})(-x_{H_k^- w} + x_{iw})(-x_{H_k^- w} + x_{jw})} \\
& +8h_b^2 h_d \mathcal{Z}_H^{1k} (\mathcal{Z}_H^{1l})^2 \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{Z}_{\tilde{U}i}^{2\alpha} \mathcal{E}^{id} \frac{x_{iw}^{\frac{3}{2}} \ln x_{H_l^- w}}{(x_{H_k^- w} - x_{iw})(-x_{H_l^- w} + x_{iw})(-x_{iw} + x_{jw})} \\
& +8h_b^2 h_d \mathcal{Z}_H^{1k} (\mathcal{Z}_H^{1l})^2 \mathcal{Z}_{\tilde{D}1}^{2\gamma} \mathcal{Z}_{\tilde{U}i}^{2\alpha} \mathcal{E}^{id} \frac{\sqrt{x_{iw}} x_{jw} \ln x_{jw}}{(x_{H_k^- w} - x_{jw})(x_{H_l^- w} - x_{jw})(x_{iw} - x_{jw})} \\
& +(i \leftrightarrow j) ,
\end{aligned} \tag{104}$$

$$\begin{aligned}
\phi_7^{hh\tilde{g}} &= \frac{8}{3 \sin \beta} h_b^2 x_{iw} \sqrt{x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{1\gamma} \mathcal{Z}_{\tilde{U}i}^{1\alpha} \mathcal{E}^{id} \left( F_D^{1a} + F_D^{1b} - F_D^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\gamma^1 w}, x_{\tilde{g}w}) \\
& + \frac{4}{3 \sin^2 \beta} h_b^2 (x_{iw} x_{jw})^{\frac{3}{2}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \left( F_C^{1a} + F_C^{1b} - F_C^{1c} \right) (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\
& - \frac{8}{3 \sin^2 \beta} h_b^2 x_{iw} x_{jw} \sqrt{x_{\tilde{g}w} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{U}i}^{1\alpha} \mathcal{Z}_{\tilde{U}i}^{2\alpha} F_C^{1a} (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w}) \\
& - \frac{16}{3 \sin \beta} h_b^2 x_{iw} \sqrt{x_{\tilde{g}w} x_{iw} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{D}1}^{1\gamma} \mathcal{Z}_{\tilde{U}i}^{2\alpha} \mathcal{E}^{id} F_D^0 (x_{iw}, x_{jw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{D}_\gamma^1 w}, x_{\tilde{g}w}) \\
& - \frac{8}{3 \sin^2 \beta} h_b^2 x_{iw}^{\frac{5}{2}} x_{jw}^{\frac{3}{2}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{1l} \mathcal{Z}_H^{2k} \mathcal{Z}_H^{2l} \mathcal{Z}_{\tilde{U}i}^{1\alpha} \mathcal{Z}_{\tilde{U}i}^{2\alpha} F_C^0 (x_{jw}, x_{iw}, x_{iw}, x_{H_k^- w}, x_{H_l^- w}, x_{\tilde{U}_\alpha^i w}, x_{\tilde{g}w})
\end{aligned}$$

$$\begin{aligned}
& -\frac{8}{3\sin\beta}h_dx_{jw}\sqrt{x_{\tilde{g}w}x_{jw}}\mathcal{Z}_H^{1k}\mathcal{Z}_H^{2i}\mathcal{Z}_{\tilde{D}^3}^{2\delta}\mathcal{Z}_{\tilde{D}^1}^{1\gamma}\mathcal{E}^{ib}\mathcal{E}^{jd}F_A^0(x_{jw},x_{H_i^-w},x_{H_k^-w},x_{\tilde{U}_\alpha^iw},x_{\tilde{D}_m^1w},x_{\tilde{D}_\gamma^1w},x_{\tilde{g}w}) \\
& +(i\leftrightarrow j),
\end{aligned} \tag{105}$$

$$\phi_8^{hh\tilde{g}} = \frac{1}{4}\phi_7^{hh\tilde{g}}, \tag{106}$$

$$\begin{aligned}
\phi_1^{sw\tilde{g}} = & \frac{4}{3}\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_{\tilde{D}^3}^{1m}\left(\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_-^{1\eta} + \frac{h_b\mathcal{Z}_{\tilde{D}^1}^{2\delta}\mathcal{Z}_-^{2\eta}}{\sqrt{2}}\right)\left(\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_-^{1\eta} + \frac{h_d\mathcal{Z}_{\tilde{D}^1}^{2\delta}\mathcal{Z}_-^{2\eta}}{\sqrt{2}}\right) \\
& \left(F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f}\right)(x_{iw},x_{jw},1,x_{\kappa_\eta^-w},x_{\tilde{g}w},x_{\tilde{D}_l^1w},x_{\tilde{D}_m^3w}) \\
& + \frac{4}{3}x_{iw}\sqrt{x_{\kappa_\eta^-w}x_{iw}}\mathcal{Z}_{\tilde{D}^3}^{1\gamma}\mathcal{Z}_{\tilde{D}^1}^{1\delta}\left(\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_-^{1\eta} + \frac{h_d\mathcal{Z}_{\tilde{D}^1}^{2\delta}\mathcal{Z}_-^{2\eta}}{\sqrt{2}}\right)\frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma}\mathcal{Z}_+^{2\eta}}{\sqrt{2}\sin\beta} \\
& \left(F_A^{1a} + F_A^{1b} - F_A^{1c}\right)(x_{iw},x_{jw},1,x_{\kappa_\eta^-w},x_{\tilde{g}w},x_{\tilde{D}_l^1w},x_{\tilde{D}_m^3w}) \\
& - \frac{4}{3}x_{iw}x_{jw}\left(\sqrt{x_{\kappa_\eta^-w}x_{jw}} + \sqrt{x_{iw}x_{jw}}\right)\mathcal{Z}_{\tilde{D}^3}^{1\gamma}\mathcal{Z}_{\tilde{D}^1}^{1\delta}\frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma}\mathcal{Z}_+^{2\eta}}{\sqrt{2}\sin\beta}\frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_+^{2\eta}}{\sqrt{2}\sin\beta} \\
& \left(F_A^{1a} + F_A^{1b} - F_A^{1c}\right)(x_{iw},x_{jw},1,x_{\kappa_\eta^-w},x_{\tilde{g}w},x_{\tilde{D}_l^1w},x_{\tilde{D}_m^3w}) \\
& + \frac{4}{3}\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{Z}_{\tilde{U}^j}^{1\beta}\left(-\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{Z}_+^{1\eta} + \frac{m_{ui}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}\mathcal{Z}_+^{2\eta}}{\sqrt{2}m_w\sin\beta}\right)\left(-\mathcal{Z}_{\tilde{U}^j}^{1\beta}\mathcal{Z}_+^{1\eta} + \frac{m_{uj}\mathcal{Z}_{\tilde{U}^j}^{2\beta}\mathcal{Z}_+^{2\eta}}{\sqrt{2}m_w\sin\beta}\right) \\
& \left(F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f}\right)(x_{iw},x_{jw},1,x_{\tilde{g}w},x_{\kappa_\eta^-w},x_{\tilde{U}_\beta^jw},x_{\tilde{U}_\alpha^iw}) \\
& + \frac{4}{3}\left(\sqrt{x_{\tilde{g}w}x_{iw}}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}\mathcal{Z}_{\tilde{U}^j}^{1\beta} + \sqrt{x_{\tilde{g}w}x_{jw}}\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{Z}_{\tilde{U}^j}^{2\beta}\right) \\
& \left(-\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{Z}_+^{1\eta} + \frac{m_{ui}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}\mathcal{Z}_+^{2\eta}}{\sqrt{2}m_w\sin\beta}\right)\left(-\mathcal{Z}_{\tilde{U}^j}^{1\beta}\mathcal{Z}_+^{1\eta} + \frac{m_{uj}\mathcal{Z}_{\tilde{U}^j}^{2\beta}\mathcal{Z}_+^{2\eta}}{\sqrt{2}m_w\sin\beta}\right) \\
& \left(F_A^{1a} + F_A^{1b} - F_A^{1c}\right)(x_{iw},x_{jw},1,x_{\tilde{g}w},x_{\kappa_\eta^-w},x_{\tilde{U}_\beta^jw},x_{\tilde{U}_\alpha^iw}) \\
& - \frac{4}{3}\sqrt{x_{iw}x_{jw}}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}\mathcal{Z}_{\tilde{U}^j}^{2\beta}\left(-\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{Z}_+^{1\eta} + \frac{m_{ui}\mathcal{Z}_{\tilde{U}^i}^{2\alpha}\mathcal{Z}_+^{2\eta}}{\sqrt{2}m_w\sin\beta}\right)\left(-\mathcal{Z}_{\tilde{U}^j}^{1\beta}\mathcal{Z}_+^{1\eta} + \frac{m_{uj}\mathcal{Z}_{\tilde{U}^j}^{2\beta}\mathcal{Z}_+^{2\eta}}{\sqrt{2}m_w\sin\beta}\right) \\
& \left(F_A^{1a} - F_A^{1b} - F_A^{1c}\right)(x_{iw},x_{jw},1,x_{\tilde{g}w},x_{\kappa_\eta^-w},x_{\tilde{U}_\beta^jw},x_{\tilde{U}_\alpha^iw}) \\
& +(i\leftrightarrow j),
\end{aligned} \tag{107}$$

$$\begin{aligned}
\phi_2^{sw\tilde{g}} = & \frac{2}{3}\left(\mathcal{Z}_{\tilde{D}^3}^{2\gamma}\mathcal{Z}_{\tilde{D}^1}^{2\delta}\left(\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_-^{1\eta} + \frac{h_b\mathcal{Z}_{\tilde{D}^1}^{2\delta}\mathcal{Z}_-^{2\eta}}{\sqrt{2}}\right)\left(\mathcal{Z}_{\tilde{D}^1}^{1\delta}\mathcal{Z}_-^{1\eta} + \frac{h_d\mathcal{Z}_{\tilde{D}^1}^{2\delta}\mathcal{Z}_-^{2\eta}}{\sqrt{2}}\right)\right. \\
& \left. + \mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{Z}_{\tilde{U}^j}^{1\beta}\frac{h_b\mathcal{Z}_{\tilde{U}^i}^{1\alpha}\mathcal{Z}_-^{2l}}{\sqrt{2}}\frac{h_d\mathcal{Z}_{\tilde{U}^j}^{1\beta}\mathcal{Z}_-^{2l}}{\sqrt{2}}\right)
\end{aligned}$$

$$\begin{aligned}
& \left( F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f} \right) (x_{iw}, x_{jw}, 1, x_{\kappa_{\eta}^- w}, x_{\tilde{g}w}, x_{\tilde{D}_l^1 w}, x_{\tilde{D}_m^3 w}) \\
& + \frac{2}{3} \left( \sqrt{x_{\tilde{g}w} x_{iw}} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} + \sqrt{x_{\tilde{g}w} x_{jw}} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \right) \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{\tilde{U}^i}^{2\alpha}}{\sqrt{2}} \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{iw}, x_{jw}, 1, x_{\tilde{g}w}, x_{\kappa_{\eta}^- w}, x_{\tilde{U}_{\beta}^j w}, x_{\tilde{U}_{\alpha}^i w}) \\
& - \frac{2}{3} \sqrt{x_{iw} x_{jw}} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{\tilde{U}^i}^{2\alpha}}{\sqrt{2}} \\
& \left( F_A^{1a} - F_A^{1b} - F_A^{1c} \right) (x_{iw}, x_{jw}, 1, x_{\tilde{g}w}, x_{\kappa_{\eta}^- w}, x_{\tilde{U}_{\beta}^j w}, x_{\tilde{U}_{\alpha}^i w}) \\
& + \frac{2}{3} x_{iw} \sqrt{x_{\kappa_{\eta}^- w} x_{iw}} \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{D}^3}^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{D}^3}^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\eta}}{\sqrt{2} \sin \beta} \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{iw}, x_{jw}, 1, x_{\kappa_{\eta}^- w}, x_{\tilde{g}w}, x_{\tilde{D}_l^1 w}, x_{\tilde{D}_m^3 w}) \\
& - \frac{2}{3} x_{iw} x_{jw} \left( \sqrt{x_{\kappa_{\eta}^- w} x_{jw}} + \sqrt{x_{iw} x_{jw}} \right) \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\eta}}{\sqrt{2} \sin \beta} \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{D}^3}^{2\eta}}{\sqrt{2} \sin \beta} \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{iw}, x_{jw}, 1, x_{\kappa_{\eta}^- w}, x_{\tilde{g}w}, x_{\tilde{D}_l^1 w}, x_{\tilde{D}_m^3 w}) \\
& + (i \leftrightarrow j) ,
\end{aligned} \tag{108}$$

$$\phi_3^{sw\tilde{g}} = -2\phi_2^{sw\tilde{g}} , \tag{109}$$

$$\begin{aligned}
\phi_1^{sh\tilde{g}} &= \frac{2}{3 \sin^2 \beta} x_{iw} x_{jw} \sqrt{x_{iw} x_{jw}} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{D}^3}^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{D}^3}^{2\eta}}{\sqrt{2}} \right) \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{D}^3}^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{D}^3}^{2\eta}}{\sqrt{2}} \right) \\
& \left( F_A^{1a} - F_A^{1b} - F_A^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_{\eta}^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& + \frac{2}{3 \sin^2 \beta} x_{iw} x_{jw} \sqrt{x_{\kappa_{\eta}^- w} x_{jw}} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( -\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{D}^3}^{1\eta} - \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{D}^3}^{2\eta}}{\sqrt{2}} \right) \frac{x_{iw} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\eta}}{\sqrt{2} \sin \beta} \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_{\eta}^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& + \frac{2}{3 \sin^2 \beta} x_{iw} \sqrt{x_{\kappa_{\eta}^- w} x_{iw} x_{jw}} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( -\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{D}^3}^{1\eta} - \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{D}^3}^{2\eta}}{\sqrt{2}} \right) \frac{x_{jw} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{D}^3}^{2\eta}}{\sqrt{2} \sin \beta} \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_{\eta}^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& - \frac{2}{3 \sin^2 \beta} x_{iw} x_{jw} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \frac{x_{iw} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\eta}}{\sqrt{2} \sin \beta} \frac{x_{jw} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{\tilde{D}^3}^{2\eta}}{\sqrt{2} \sin \beta} \\
& \left( F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_{\eta}^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& - \frac{2}{3 \sin^2 \beta} x_{iw} x_{jw} \sqrt{x_{iw} x_{jw}} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\eta} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{\tilde{U}^i}^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_{\tilde{U}^i}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right)
\end{aligned}$$



$$\begin{aligned}
& (F_A^{1a} - F_A^{1b} - F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_{\eta}^- w}, x_{\tilde{U}_{\beta}^j w}, x_{\tilde{U}_{\alpha}^i w}) \\
& + \frac{2}{3 \sin^2 \beta} x_{iw} x_{jw} \sqrt{x_{\tilde{g}w} x_{jw}} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& (F_A^{1a} + F_A^{1b} - F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_{\eta}^- w}, x_{\tilde{U}_{\beta}^j w}, x_{\tilde{U}_{\alpha}^i w}) \\
& + \frac{2}{3 \sin^2 \beta} x_{iw} x_{jw} \sqrt{x_{\tilde{g}w} x_{iw}} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& (F_A^{1a} + F_A^{1b} - F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_{\eta}^- w}, x_{\tilde{U}_{\beta}^j w}, x_{\tilde{U}_{\alpha}^i w}) \\
& + \frac{2}{3 \sin^2 \beta} x_{iw} x_{jw} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& (F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_{\eta}^- w}, x_{\tilde{U}_{\beta}^j w}, x_{\tilde{U}_{\alpha}^i w}) \\
& + (i \leftrightarrow j), \tag{110}
\end{aligned}$$

$$\begin{aligned}
\phi_2^{sh\tilde{g}} = & -\frac{1}{3} h_b h_d (\mathcal{Z}_H^{1k})^2 \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \\
& (F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_{\eta}^- w}, x_{\tilde{g}w}, x_{\tilde{D}_{\delta}^1 w}, x_{\tilde{D}_{\gamma}^3 w}) \\
& - \frac{2}{3 \sin \beta} h_d x_{iw} \sqrt{x_{\tilde{g}w} x_{iw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_{\eta}^- w}, x_{\tilde{g}w}, x_{\tilde{D}_{\delta}^1 w}, x_{\tilde{D}_{\gamma}^3 w}) \\
& - \frac{4}{3 \sin \beta} h_b x_{jw} \sqrt{x_{\tilde{g}w} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_{\eta}^- w}, x_{\tilde{g}w}, x_{\tilde{D}_{\delta}^1 w}, x_{\tilde{D}_{\gamma}^3 w}) \\
& + \frac{1}{3 \sin^2 \beta} (x_{iw} x_{jw})^{\frac{3}{2}} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \\
& (F_A^{1a} - F_A^{1b} - F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_{\eta}^- w}, x_{\tilde{g}w}, x_{\tilde{D}_{\delta}^1 w}, x_{\tilde{D}_{\gamma}^3 w}) \\
& - \frac{1}{3 \sin^2 \beta} (x_{iw} x_{jw})^{\frac{3}{2}} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \\
& (F_A^{1a} - F_A^{1b} - F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_{\eta}^- w}, x_{\tilde{U}_{\beta}^j w}, x_{\tilde{U}_{\alpha}^i w}) \\
& + \frac{1}{2 \sin^2 \beta} x_{iw} x_{jw} \sqrt{x_{\tilde{g}w} x_{jw}} (\mathcal{Z}_H^{2k})^2 \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \right) \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \\
& (F_A^{1a} + F_A^{1b} - F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_{\eta}^- w}, x_{\tilde{U}_{\beta}^j w}, x_{\tilde{U}_{\alpha}^i w})
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{3 \sin^2 \beta} (x_{iw} x_{jw} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \\
& (F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f}) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_\eta^- w}, x_{\tilde{U}_\beta^j w}, x_{\tilde{U}_\alpha^i w}) \\
& - \frac{1}{3} h_b h_d \sqrt{x_{\kappa_\eta^- w} x_{iw}} (\mathcal{Z}_H^{1k})^2 \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{x_{iw} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& (F_A^{1a} + F_A^{1b} - F_A^{1c}) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& + \frac{2}{3 \sin \beta} h_d \sqrt{x_{\kappa_\eta^- w} x_{\tilde{g}w}} x_{iw}^2 \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& F_A^{1a} (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& + \frac{4}{3 \sin \beta} h_b x_{jw} x_{iw} \sqrt{x_{\kappa_\eta^- w} x_{\tilde{g}w} x_{iw} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& F_A^0 (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& - \frac{1}{3 \sin^2 \beta} x_{iw}^2 x_{jw} \sqrt{x_{\kappa_\eta^- w} x_{jw}} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& (F_A^{1a} + F_A^{1b} - F_A^{1c}) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& - \frac{1}{3} h_b h_d x_{jw} \sqrt{x_{\kappa_\eta^- w} x_{jw}} (\mathcal{Z}_H^{1k})^2 \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& (F_A^{1a} + F_A^{1b} - F_A^{1c}) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& + \frac{4}{3 \sin \beta} h_d x_{iw} x_{jw} \sqrt{x_{\kappa_\eta^- w} x_{\tilde{g}w} x_{iw} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& F_A^0 (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& + \frac{2}{3 \sin \beta} h_b x_{jw} \sqrt{x_{\kappa_\eta^- w} x_{\tilde{g}w} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& F_A^{1a} (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& - \frac{1}{3 \sin^2 \beta} x_{iw} x_{jw}^2 \sqrt{x_{\kappa_\eta^- w} x_{iw}} (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& (F_A^{1a} + F_A^{1b} - F_A^{1c}) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& + \frac{1}{3} h_b h_d (x_{iw} x_{jw})^{\frac{3}{2}} (\mathcal{Z}_H^{1k})^2 \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta}
\end{aligned}$$

$$\begin{aligned}
& (F_A^{1a} - F_A^{1b} - F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& - \frac{4}{3 \sin \beta} h_d x_{iw}^2 x_{jw} \sqrt{x_{\tilde{g}w} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& - \frac{2}{3 \sin \beta} h_b x_{iw} x_{jw}^2 \sqrt{x_{\tilde{g}w} x_{iw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& - \frac{1}{3 \sin^2 \beta} x_{iw}^2 x_{jw}^2 (\mathcal{Z}_H^{2k})^2 \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& (F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& - \frac{2}{3 \sin \beta} h_b x_{iw} \sqrt{x_{\kappa_\eta^- w} x_{iw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_\eta^- w}, x_{\tilde{U}_\beta^j w}, x_{\tilde{U}_\alpha^i w}) \\
& + \frac{4}{3 \sin \beta} h_b x_{iw} \sqrt{x_{\tilde{g}w} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_A^{1a}(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_\eta^- w}, x_{\tilde{U}_\beta^j w}, x_{\tilde{U}_\alpha^i w}) \\
& + \frac{4}{3 \sin \beta} h_b x_{iw} \sqrt{x_{\kappa_i^- w} x_{\kappa_\eta^- w} x_{\tilde{g}w} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_A^0(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_\eta^- w}, x_{\tilde{U}_\beta^j w}, x_{\tilde{U}_\alpha^i w}) \\
& - \frac{2}{3 \sin \beta} h_b x_{iw} \sqrt{x_{\kappa_\eta^- w} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_\eta^- w}, x_{\tilde{U}_\beta^j w}, x_{\tilde{U}_\alpha^i w}) \\
& - \frac{2}{3 \sin \beta} h_d x_{jw} \sqrt{x_{\kappa_\eta^- w} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_\eta^- w}, x_{\tilde{U}_\beta^j w}, x_{\tilde{U}_\alpha^i w}) \\
& + \frac{4}{3 \sin \beta} h_d x_{jw} \sqrt{x_{\kappa_i^- w} x_{\kappa_\eta^- w} x_{\tilde{g}w} x_{lw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_A^0(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_\eta^- w}, x_{\tilde{U}_\beta^j w}, x_{\tilde{U}_\alpha^i w})
\end{aligned}$$

$$\begin{aligned}
& + \frac{4}{3 \sin \beta} h_d x_{jw} \sqrt{x_{\tilde{g}w} x_{lw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_A^{1a}(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_{\eta^-} w}, x_{\tilde{U}_{\beta}^j w}, x_{\tilde{U}_{\alpha}^i w}) \\
& - \frac{2}{3 \sin \beta} h_d \sqrt{x_{\kappa_{\eta^-} w} x_{iw} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( F_A^{1a} - F_A^{1b} + F_A^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_{\eta^-} w}, x_{\tilde{U}_{\beta}^j w}, x_{\tilde{U}_{\alpha}^i w}) \\
& + \frac{1}{3} h_b h_d (\mathcal{Z}_H^{1k})^2 \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_{\eta^-} w}, x_{\tilde{U}_{\beta}^j w}, x_{\tilde{U}_{\alpha}^i w}) \\
& + \frac{1}{3} h_b h_d \sqrt{x_{\tilde{g}w} x_{iw}} (\mathcal{Z}_H^{1k})^2 \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} - \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \right) \\
& \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_{\eta^-} w}, x_{\tilde{U}_{\beta}^j w}, x_{\tilde{U}_{\alpha}^i w}) \\
& +(i \leftrightarrow j) ,
\end{aligned} \tag{111}$$

$$\phi_3^{sh\tilde{g}} = -2\phi_2^{sh\tilde{g}} , \tag{112}$$

$$\begin{aligned}
\phi_4^{sh\tilde{g}} &= \frac{2}{3 \sin \beta} h_d x_{iw} \sqrt{x_{\tilde{g}w} x_{iw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \\
& \left( F_A^{1a} - F_A^{1b} + F_A^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_{\eta^-} w}, x_{\tilde{g}w}, x_{\tilde{D}_{\delta}^1 w}, x_{\tilde{D}_{\gamma}^3 w}) \\
& - \frac{2}{3 \sin \beta} h_d x_{iw}^2 \sqrt{x_{\kappa_{\eta^-} w} x_{\tilde{g}w}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& F_A^{1a}(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_{\eta^-} w}, x_{\tilde{g}w}, x_{\tilde{D}_{\delta}^1 w}, x_{\tilde{D}_{\gamma}^3 w}) \\
& - \frac{4}{3 \sin \beta} h_d (x_{iw} x_{jw})^{\frac{3}{2}} \sqrt{x_{\kappa_{\eta^-} w} x_{\tilde{g}w}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& F_A^0(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_{\eta^-} w}, x_{\tilde{g}w}, x_{\tilde{D}_{\delta}^1 w}, x_{\tilde{D}_{\gamma}^3 w}) \\
& + \frac{4}{3 \sin \beta} h_d x_{iw}^2 x_{jw} \sqrt{x_{\tilde{g}w} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& \left( F_A^{1a} - F_A^{1b} + F_A^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_{\eta^-} w}, x_{\tilde{g}w}, x_{\tilde{D}_{\delta}^1 w}, x_{\tilde{D}_{\gamma}^3 w})
\end{aligned}$$

$$\begin{aligned}
& + \frac{2}{3 \sin \beta} h_d x_{jw} \sqrt{x_{\kappa_{\eta^-}^-} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{\tilde{U}^i}^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( F_A^{1a} - F_A^{1b} + F_A^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^-}, x_{\tilde{g}w}, x_{\kappa_{\eta^-}^-}, x_{\tilde{U}_{\beta}^j}, x_{\tilde{U}_{\alpha}^i}) \\
& - \frac{4}{3 \sin \beta} h_d x_{jw} \sqrt{x_{\kappa_{\eta^-}^-} x_{\kappa_{\eta^-}^-} x_{\tilde{g}w} x_{lw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{\tilde{U}^i}^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_A^0(x_{iw}, x_{jw}, x_{H_k^-}, x_{\tilde{g}w}, x_{\kappa_{\eta^-}^-}, x_{\tilde{U}_{\beta}^j}, x_{\tilde{U}_{\alpha}^i}) \\
& - \frac{4}{3 \sin \beta} h_d x_{jw} \sqrt{x_{\tilde{g}w} x_{lw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{\tilde{U}^i}^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_A^{1a}(x_{iw}, x_{jw}, x_{H_k^-}, x_{\tilde{g}w}, x_{\kappa_{\eta^-}^-}, x_{\tilde{U}_{\beta}^j}, x_{\tilde{U}_{\alpha}^i}) \\
& + \frac{2}{3 \sin \beta} h_d \sqrt{x_{\kappa_{\eta^-}^-} x_{iw} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{\tilde{U}^i}^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( F_A^{1a} - F_A^{1b} + F_A^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^-}, x_{\tilde{g}w}, x_{\kappa_{\eta^-}^-}, x_{\tilde{U}_{\beta}^j}, x_{\tilde{U}_{\alpha}^i}) \\
& + (i \leftrightarrow j), \tag{113}
\end{aligned}$$

$$\phi_5^{sh\tilde{g}} = \frac{1}{4} \phi_4^{sh\tilde{g}}, \tag{114}$$

$$\begin{aligned}
\phi_6^{sh\tilde{g}} &= -\frac{2}{3} h_b h_d (\mathcal{Z}_H^{1k})^2 \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \\
& \left( F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f} \right) (x_{iw}, x_{jw}, x_{H_k^-}, x_{\kappa_{\eta^-}^-}, x_{\tilde{g}w}, x_{\tilde{D}_{\delta}^1}, x_{\tilde{D}_{\gamma}^3}) \\
& + \frac{2}{3} h_b h_d (\mathcal{Z}_H^{1k})^2 \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \\
& \left( F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f} \right) (x_{iw}, x_{jw}, x_{H_k^-}, x_{\tilde{g}w}, x_{\kappa_{\eta^-}^-}, x_{\tilde{U}_{\beta}^j}, x_{\tilde{U}_{\alpha}^i}) \\
& + \frac{2}{3} h_b h_d \sqrt{x_{\tilde{g}w} x_{iw}} (\mathcal{Z}_H^{1k})^2 \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} - \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \right) \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^-}, x_{\tilde{g}w}, x_{\kappa_{\eta^-}^-}, x_{\tilde{U}_{\beta}^j}, x_{\tilde{U}_{\alpha}^i}) \\
& - \frac{2}{3} h_b h_d x_{iw} \sqrt{x_{\kappa_{\eta^-}^-} x_{iw}} (\mathcal{Z}_H^{1k})^2 \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{iw}, x_{jw}, x_{H_k^-}, x_{\kappa_{\eta^-}^-}, x_{\tilde{g}w}, x_{\tilde{D}_{\delta}^1}, x_{\tilde{D}_{\gamma}^3}) \\
& - \frac{2}{3} h_b h_d x_{jw} \sqrt{x_{\kappa_{\eta^-}^-} x_{jw}} (\mathcal{Z}_H^{1k})^2 \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta}
\end{aligned}$$

$$\begin{aligned}
& (F_A^{1a} + F_A^{1b} - F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& + \frac{2}{3} h_b h_d (x_{iw} x_{jw})^{\frac{3}{2}} (\mathcal{Z}_H^{1k})^2 \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{2\delta} \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& (F_A^{1a} - F_A^{1b} - F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& + (i \leftrightarrow j) ,
\end{aligned} \tag{115}$$

$$\begin{aligned}
\phi_7^{sh\tilde{g}} &= \frac{4}{3 \sin \beta} h_b x_{jw} \sqrt{x_{\tilde{g}w} x_{jw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& - \frac{4}{3 \sin \beta} h_b (x_{iw} x_{jw})^2 \sqrt{x_{\kappa_\eta^- w} x_{\tilde{g}w}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& F_A^0(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& - \frac{2}{3 \sin \beta} h_b x_{jw}^2 \sqrt{x_{\kappa_\eta^- w} x_{\tilde{g}w}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta} + \frac{h_b \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& F_A^{1a}(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& + \frac{2}{3 \sin \beta} h_b x_{iw} x_{jw}^2 \sqrt{x_{\tilde{g}w} x_{iw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \frac{\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \frac{\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} \sin \beta} \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\kappa_\eta^- w}, x_{\tilde{g}w}, x_{\tilde{D}_\delta^1 w}, x_{\tilde{D}_\gamma^3 w}) \\
& + \frac{2}{3 \sin \beta} h_b x_{iw} \sqrt{x_{\kappa_\eta^- w} x_{iw}} \mathcal{Z}_H^{1k} \mathcal{Z}_H^{2k} \left( \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} - 2 \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{1\beta} + \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \right) \\
& \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \left( - \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{iw}, x_{jw}, x_{H_k^- w}, x_{\tilde{g}w}, x_{\kappa_\eta^- w}, x_{\tilde{U}_\beta^j w}, x_{\tilde{U}_\alpha^i w}) \\
& + (i \leftrightarrow j) ,
\end{aligned} \tag{116}$$

$$\phi_8^{sh\tilde{g}} = \frac{1}{4} \phi_7^{sh\tilde{g}} , \tag{117}$$

$$\begin{aligned}
\phi_1^{p\tilde{g}} &= -\frac{16}{3} \left( \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( - \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_-^{1\lambda} + \frac{h_b \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_-^{2\lambda}}{\sqrt{2}} \right) \left( - (\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_-^{1\eta}) + \frac{(h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_-^{2\eta})}{\sqrt{2}} \right) \right. \\
& \left. \left( - (\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\lambda}) + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( - \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& (F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f})(x_{\kappa_{\lambda}^- w}, x_{\tilde{U}_{\alpha}^i w}, x_{\kappa_{\eta}^- w}, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_{\gamma}^3 w}, x_{\tilde{D}_{\delta}^1 w}) \\
& + \frac{16}{3} \sqrt{x_{\kappa_{\lambda}^- w} x_{jw}} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( -\mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{-}^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2\delta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \right) \frac{m_{uj} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \\
& \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& (F_A^{1a} + F_A^{1b} - F_A^{1c})(x_{\kappa_{\lambda}^- w}, x_{\tilde{U}_{\alpha}^i w}, x_{\kappa_{\eta}^- w}, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_{\gamma}^3 w}, x_{\tilde{D}_{\delta}^1 w}) \\
& + \frac{16}{3} \sqrt{x_{\kappa_{\eta}^- w} x_{jw}} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \left( -\mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{-}^{1\lambda} + \frac{h_b \mathcal{Z}_{\tilde{D}^3}^{2\gamma} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \right) \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& \frac{m_{uj} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& (F_A^{1a} + F_A^{1b} - F_A^{1c})(x_{\kappa_{\lambda}^- w}, x_{\tilde{U}_{\alpha}^i w}, x_{\kappa_{\eta}^- w}, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_{\gamma}^3 w}, x_{\tilde{D}_{\delta}^1 w}) \\
& + \frac{16}{3} \sqrt{x_{\kappa_{\lambda}^- w} x_{\kappa_{\eta}^- w}} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \frac{m_{uj} \mathcal{Z}_{\tilde{D}^3}^{1\gamma} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \left( -(\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\lambda}) + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& \frac{m_{uj} \mathcal{Z}_{\tilde{D}^1}^{1\delta} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \left( -(\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\eta}) + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& (F_A^{1a} - F_A^{1b} - F_A^{1c})(x_{\kappa_{\lambda}^- w}, x_{\tilde{U}_{\alpha}^i w}, x_{\kappa_{\eta}^- w}, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_{\gamma}^3 w}, x_{\tilde{D}_{\delta}^1 w}) \\
& - \frac{64}{3} \sqrt{x_{\tilde{g}w} x_{iw}} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \left( -(\mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{+}^{1\lambda}) + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_{+}^{1\lambda} \right. \\
& \left. + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{+}^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{+}^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_C^{1a}(x_{\tilde{U}_{\alpha}^i w}, x_{\kappa_{\lambda}^- w}, x_{\kappa_{\eta}^- w}, x_{\tilde{U}_{\beta}^j w}, x_{iw}, x_{\tilde{U}_{\alpha}^i w}, x_{\tilde{g}w}) \\
& - 32 x_{\kappa_{\lambda}^- w} \sqrt{x_{\tilde{g}w} x_{iw}} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^m}^{1n} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^m}^{2n} \right) \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{+}^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{+}^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( \frac{\ln x_{\kappa_{\lambda}^- w}}{(-x_{\kappa_{\lambda}^- w} + x_{\kappa_{\eta}^- w})(-x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\alpha}^i w})(-x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\beta}^j w})} \right. \\
& \left. - \frac{x_{iw} \ln x_{\kappa_{\lambda}^- w}}{(-x_{\kappa_{\lambda}^- w} + x_{\kappa_{\eta}^- w})(-x_{\kappa_{\lambda}^- w} + x_{iw})(-x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\alpha}^i w})(-x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\beta}^j w})} \right) \\
& - 32 x_{\kappa_{\eta}^- w} \sqrt{x_{\tilde{g}w} x_{iw}} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -(\mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_+^{1\lambda}) + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \mathcal{Z}_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( \frac{\ln x_{\kappa_\eta^- w}}{(x_{\kappa_\lambda^- w} - x_{\kappa_\eta^- w})(-x_{\kappa_\eta^- w} + x_{\tilde{U}_\alpha^i w})(-x_{\kappa_\eta^- w} + x_{\tilde{U}_\beta^j w})} \right. \\
& \left. - \frac{x_{iw} \ln x_{\kappa_\eta^- w}}{(x_{\kappa_\lambda^- w} - x_{\kappa_\eta^- w})(-x_{\kappa_\eta^- w} + x_{iw})(-x_{\kappa_\eta^- w} + x_{\tilde{U}_\alpha^i w})(-x_{\kappa_\eta^- w} + x_{\tilde{U}_\beta^j w})} \right) \\
& + 32 x_{iw}^2 \sqrt{x_{\tilde{g}w} x_{iw}} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^m}^{1n} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_+^{1\lambda} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_+^{1\lambda} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \mathcal{Z}_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( \frac{\ln x_{iw}}{(x_{\kappa_\lambda^- w} - x_{iw})(x_{\kappa_\eta^- w} - x_{iw})(-x_{iw} + x_{\tilde{U}_\alpha^i w})(-x_{iw} + x_{\tilde{U}_\beta^j w})} \right. \\
& \left. - 32(\sqrt{x_{\tilde{g}w} x_{iw}} x_{\tilde{U}_\alpha^i w} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_+^{1\lambda} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \right. \\
& \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_+^{1\lambda} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \mathcal{Z}_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( \frac{\ln x_{\tilde{U}_\alpha^i w}}{(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\alpha^i w})(x_{\kappa_\eta^- w} - x_{\tilde{U}_\alpha^i w})(-x_{\tilde{U}_\alpha^i w} + x_{\tilde{U}_\beta^j w})} \right. \\
& \left. - \frac{x_{iw} \ln x_{\tilde{U}_\alpha^i w}}{(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\alpha^i w})(x_{\kappa_\eta^- w} - x_{\tilde{U}_\alpha^i w})(x_{iw} - x_{\tilde{U}_\alpha^i w})(-x_{\tilde{U}_\alpha^i w} + x_{\tilde{U}_\beta^j w})} \right) \\
& - 32 \sqrt{x_{\tilde{g}w} x_{iw}} x_{\tilde{U}_\beta^j w} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^m}^{1i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_+^{1\lambda} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_+^{1\lambda} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \mathcal{Z}_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( \frac{\ln x_{\tilde{U}_\beta^j w}}{(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\beta^j w})(x_{\kappa_\eta^- w} - x_{\tilde{U}_\beta^j w})(x_{\tilde{U}_\alpha^i w} - x_{\tilde{U}_\beta^j w})} \right. \\
& \left. - \frac{x_{iw} \ln x_{\tilde{U}_\beta^j w}}{(x_{\kappa_\lambda^- w} - x_{\tilde{U}_\beta^j w})(x_{\kappa_\eta^- w} - x_{\tilde{U}_\beta^j w})(x_{iw} - x_{\tilde{U}_\beta^j w})(x_{\tilde{U}_\alpha^i w} - x_{\tilde{U}_\beta^j w})} \right) + (i \leftrightarrow j), \tag{118}
\end{aligned}$$

$$\begin{aligned}
\phi_2^{p\bar{g}} &= 8 \sqrt{x_{\kappa_\lambda^- w} x_{\kappa_\eta^- w}} \mathcal{Z}_{\tilde{D}^3}^{1n} \mathcal{Z}_{\tilde{D}^1}^{1m} \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2\lambda}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_-^{1\lambda} + \frac{h_b \mathcal{Z}_{\tilde{D}^3}^{2m} \mathcal{Z}_-^{2\lambda}}{\sqrt{2}} \right) \\
& \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2n} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right)
\end{aligned}$$



$$\begin{aligned}
& \left( F_A^{1a} - F_A^{1b} - F_A^{1c} \right) (x_{\kappa_\lambda^- w}, x_{\tilde{U}_w^i}, x_{\kappa_\eta^- w}, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_{m^3} w}, x_{\tilde{D}_{n^1} w}) \\
& + \frac{8}{3} \sqrt{x_{\kappa_\eta^- w} x_{jw}} Z_{\tilde{D}^3}^{1n} Z_{\tilde{D}^1}^{1m} \frac{h_d Z_{\tilde{U}^i}^{1\alpha} Z_-^{2\lambda}}{\sqrt{2}} \frac{h_b Z_{\tilde{U}^i}^{1\alpha} Z_-^{2\eta}}{\sqrt{2}} \left( -Z_{\tilde{D}^1}^{1n} Z_-^{1\eta} + \frac{h_d Z_{\tilde{D}^1}^{2n} Z_-^{2\eta}}{\sqrt{2}} \right) \frac{m_{uj} Z_{\tilde{D}^3}^{1m} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{\kappa_\lambda^- w}, x_{\tilde{U}_w^i}, x_{\kappa_\eta^- w}, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_{m^3} w}, x_{\tilde{D}_{n^1} w}) \\
& + \frac{8}{3} \sqrt{x_{\kappa_\lambda^- w} x_{jw}} Z_{\tilde{D}^3}^{1n} Z_{\tilde{D}^1}^{1m} \frac{h_d Z_{\tilde{U}^i}^{1\alpha} Z_-^{2\lambda}}{\sqrt{2}} \left( -Z_{\tilde{D}^3}^{1m} Z_-^{1\lambda} + \frac{h_b Z_{\tilde{D}^3}^{2m} Z_-^{2\lambda}}{\sqrt{2}} \right) \frac{h_b Z_{\tilde{U}^i}^{1\alpha} Z_-^{2\eta}}{\sqrt{2}} \frac{m_{uj} Z_{\tilde{D}^1}^{1n} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{\kappa_\lambda^- w}, x_{\tilde{U}_w^i}, x_{\kappa_\eta^- w}, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_{m^3} w}, x_{\tilde{D}_{n^1} w}) \\
& - \frac{8}{3} Z_{\tilde{D}^3}^{1n} Z_{\tilde{D}^1}^{1m} \frac{h_d Z_{\tilde{U}^i}^{1j} Z_-^{2\lambda}}{\sqrt{2}} \frac{h_b Z_{\tilde{U}^i}^{1\alpha} Z_-^{2\eta}}{\sqrt{2}} \frac{m_{uj} Z_{\tilde{D}^3}^{1m} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \frac{m_{uj} Z_{\tilde{D}^1}^{1n} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \\
& \left( F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f} \right) (x_{\kappa_\lambda^- w}, x_{\tilde{U}_w^i}, x_{\kappa_\eta^- w}, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_{m^3} w}, x_{\tilde{D}_{n^1} w}) \\
& - \frac{8}{3} Z_{\tilde{D}^3}^{2n} Z_{\tilde{D}^1}^{2m} \left( -Z_{\tilde{D}^3}^{1m} Z_-^{1\lambda} + \frac{h_b Z_{\tilde{D}^3}^{2m} Z_-^{2\lambda}}{\sqrt{2}} \right) \left( -Z_{\tilde{D}^1}^{1n} Z_-^{1\eta} + \frac{h_d Z_{\tilde{D}^1}^{2n} Z_-^{2\eta}}{\sqrt{2}} \right) \\
& \left( -Z_{\tilde{U}^i}^{1\alpha} Z_+^{1\lambda} + \frac{m_{ui} Z_{\tilde{U}^i}^{2\alpha} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -Z_{\tilde{U}^i}^{1\alpha} Z_+^{1\eta} + \frac{m_{uj} Z_{\tilde{U}^i}^{2\alpha} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f} \right) (x_{\kappa_\lambda^- w}, x_{\tilde{U}_w^i}, x_{\kappa_\eta^- w}, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_{m^3} w}, x_{\tilde{D}_{n^1} w}) \\
& + \frac{8}{3} \sqrt{x_{\kappa_\lambda^- w} x_{jw}} Z_{\tilde{D}^3}^{2n} Z_{\tilde{D}^1}^{2m} \left( -Z_{\tilde{D}^1}^{1n} Z_-^{1\eta} + \frac{h_d Z_{\tilde{D}^1}^{2n} Z_-^{2\eta}}{\sqrt{2}} \right) \frac{m_{uj} Z_{\tilde{D}^3}^{1m} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \\
& \left( -Z_{\tilde{U}^i}^{1\alpha} Z_+^{1\lambda} + \frac{m_{ui} Z_{\tilde{U}^i}^{2\alpha} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -Z_{\tilde{U}^i}^{1\alpha} Z_+^{1\eta} + \frac{m_{uj} Z_{\tilde{U}^i}^{2\alpha} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{\kappa_\lambda^- w}, x_{\tilde{U}_w^i}, x_{\kappa_\eta^- w}, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_{m^3} w}, x_{\tilde{D}_{n^1} w}) \\
& + \frac{8}{3} \sqrt{x_{\kappa_\eta^- w} x_{jw}} Z_{\tilde{D}^3}^{2n} Z_{\tilde{D}^1}^{2m} \left( -Z_{\tilde{D}^3}^{1m} Z_-^{1\lambda} + \frac{h_b Z_{\tilde{D}^3}^{2m} Z_-^{2\lambda}}{\sqrt{2}} \right) \left( -Z_{\tilde{U}^i}^{1\alpha} Z_+^{1\lambda} + \frac{m_{ui} Z_{\tilde{U}^i}^{2\alpha} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& \frac{m_{uj} Z_{\tilde{D}^1}^{1n} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \left( -Z_{\tilde{U}^i}^{1\alpha} Z_+^{1\eta} + \frac{m_{uj} Z_{\tilde{U}^i}^{2\alpha} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{\kappa_\lambda^- w}, x_{\tilde{U}_w^i}, x_{\kappa_\eta^- w}, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_{m^3} w}, x_{\tilde{D}_{n^1} w}) \\
& + \frac{8}{3} \sqrt{x_{\kappa_\lambda^- w} x_{\kappa_\eta^- w}} Z_{\tilde{D}^3}^{2n} Z_{\tilde{D}^1}^{2m} \frac{m_{uj} Z_{\tilde{D}^3}^{1m} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \left( -Z_{\tilde{U}^i}^{1\alpha} Z_+^{1\lambda} + \frac{m_{ui} Z_{\tilde{U}^i}^{2\alpha} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& \frac{m_{uj} Z_{\tilde{D}^1}^{1n} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \left( -Z_{\tilde{U}^i}^{1\alpha} Z_+^{1\eta} + \frac{m_{uj} Z_{\tilde{U}^i}^{2\alpha} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( F_A^{1a} - F_A^{1b} - F_A^{1c} \right) (x_{\kappa_\lambda^- w}, x_{\tilde{U}_w^i}, x_{\kappa_\eta^- w}, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_{m^3} w}, x_{\tilde{D}_{n^1} w})
\end{aligned}$$

$$\begin{aligned}
& + \frac{32}{3} \sqrt{x_{\kappa_{\lambda}^-} x_{\kappa_{\eta}^-} x_{\tilde{g}w} x_{jw}} \mathcal{Z}_{\tilde{D}^3}^{1n} \mathcal{Z}_{\tilde{D}^1}^{2m} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{-}^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2n} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \right) \\
& \frac{m_{uj} \mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_A^0(x_{\kappa_{\lambda}^-} w, x_{\tilde{U}_j^I} w, x_{\kappa_{\eta}^-} w, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_m^3} w, x_{\tilde{D}_n^1} w) \\
& - \frac{32}{3} \sqrt{x_{\tilde{g}w} x_{iw}} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_b \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \\
& \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_C^{1a}(x_{\tilde{U}_\alpha^i} w, x_{\kappa_{\lambda}^-} w, x_{\kappa_{\eta}^-} w, x_{\tilde{U}_l^I} w, x_{jw}, x_{\tilde{U}_\beta^j} w, x_{\tilde{g}w}) \\
& + \frac{32}{3} \sqrt{x_{\tilde{g}w} x_{jw}} \mathcal{Z}_{\tilde{D}^3}^{1n} \mathcal{Z}_{\tilde{D}^1}^{2m} \left( -\mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{-}^{1\lambda} + \frac{h_b \mathcal{Z}_{\tilde{D}^3}^{2m} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \right) \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \\
& \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \frac{m_{uj} \mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \\
& F_A^{1a}(x_{\kappa_{\lambda}^-} w, x_{\tilde{U}_j^I} w, x_{\kappa_{\eta}^-} w, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_m^3} w, x_{\tilde{D}_n^1} w) \\
& + \frac{16}{3} \sqrt{x_{\kappa_{\lambda}^-} w x_{\tilde{g}w}} \left( \mathcal{Z}_{\tilde{D}^3}^{1n} \mathcal{Z}_{\tilde{D}^1}^{2m} + \mathcal{Z}_{\tilde{D}^3}^{2n} \mathcal{Z}_{\tilde{D}^1}^{1m} \right) \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \frac{m_{uj} \mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \\
& \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{ui} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \frac{m_{uj} \mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{\kappa_{\lambda}^-} w, x_{\tilde{U}_j^I} w, x_{\kappa_{\eta}^-} w, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_m^3} w, x_{\tilde{D}_n^1} w) \\
& + \frac{32}{3} \sqrt{x_{\tilde{g}w} x_{jw}} \mathcal{Z}_{\tilde{D}^3}^{2n} \mathcal{Z}_{\tilde{D}^1}^{1m} \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{-}^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2n} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \right) \\
& \frac{m_{uj} \mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_A^{1a}(x_{\kappa_{\lambda}^-} w, x_{\tilde{U}_j^I} w, x_{\kappa_{\eta}^-} w, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_m^3} w, x_{\tilde{D}_n^1} w) \\
& + \frac{32}{3} \sqrt{x_{\kappa_{\lambda}^-} w x_{\kappa_{\eta}^-} w x_{\tilde{g}w} x_{jw}} \mathcal{Z}_{\tilde{D}^3}^{2n} \mathcal{Z}_{\tilde{D}^1}^{1m} \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{-}^{1\lambda} + \frac{h_b \mathcal{Z}_{\tilde{D}^3}^{2m} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \right) \\
& \frac{m_{uj} \mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_A^0(x_{\kappa_{\lambda}^-} w, x_{\tilde{U}_j^I} w, x_{\kappa_{\eta}^-} w, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_m^3} w, x_{\tilde{D}_n^1} w)
\end{aligned}$$

$$\begin{aligned}
& + \frac{16}{3} \sqrt{x_{\kappa_{\eta}^-} w} x_{\tilde{g}w} Z_{\tilde{D}^3}^{2n} Z_{\tilde{D}^1}^{1m} \frac{h_d Z_{\tilde{U}^i}^{1\alpha} Z_-^{2\lambda}}{\sqrt{2}} \frac{m_{uj} Z_{\tilde{D}^3}^{1m} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \frac{m_{uj} Z_{\tilde{D}^1}^{1n} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \left( -Z_{\tilde{U}^i}^{1\alpha} Z_+^{1\eta} + \frac{m_{uj} Z_{\tilde{U}^i}^{2\alpha} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( F_A^{1a} - F_A^{1b} + F_A^{1c} \right) (x_{\kappa_{\lambda}^-} w, x_{\tilde{U}_j^I} w, x_{\kappa_{\eta}^-} w, x_{jw}, x_{\tilde{g}w}, x_{\tilde{D}_m^3} w, x_{\tilde{D}_n^1} w) \\
& - \frac{32}{3} \sqrt{x_{\tilde{g}w} x_{iw}} \left( Z_{\tilde{U}^i}^{2\alpha_1} Z_{\tilde{U}^i}^{1\alpha_2} + Z_{\tilde{U}^i}^{1\alpha_1} Z_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_d Z_{\tilde{U}^i}^{1\alpha_1} Z_-^{2\lambda}}{\sqrt{2}} \frac{h_b Z_{\tilde{U}^i}^{1\alpha_2} Z_-^{2\lambda}}{\sqrt{2}} \\
& \left( -Z_{\tilde{U}^j}^{1\beta} Z_+^{1\eta} + \frac{m_{uj} Z_{\tilde{U}^j}^{2\beta} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -Z_{\tilde{U}^j}^{1\beta} Z_+^{1\eta} + \frac{m_{uj} Z_{\tilde{U}^j}^{2\beta} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_C^{1a} (x_{\tilde{U}_{\alpha}^i} w, x_{\kappa_{\lambda}^-} w, x_{\kappa_{\eta}^-} w, x_{\tilde{U}_l^I} w, x_{mw}, x_{\tilde{U}_{\beta}^j} w, x_{\tilde{g}w}) \\
& + \frac{16}{3} x_{\kappa_{\lambda}^-} w \sqrt{x_{\tilde{g}w} x_{iw}} \left( Z_{\tilde{U}^i}^{2\alpha_1} Z_{\tilde{U}^i}^{1\alpha_2} + Z_{\tilde{U}^i}^{1\alpha_1} Z_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_b Z_{\tilde{U}^j}^{1\beta} Z_-^{2\eta}}{\sqrt{2}} \frac{h_d Z_{\tilde{U}^j}^{1\beta} Z_-^{2\eta}}{\sqrt{2}} \\
& \left( -Z_{\tilde{U}^i}^{1\alpha_1} Z_+^{1\lambda} + \frac{m_{ui} Z_{\tilde{U}^i}^{1\alpha_1} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -Z_{\tilde{U}^i}^{1\alpha_2} Z_+^{1\lambda} + \frac{m_{ui} Z_{\tilde{U}^i}^{2\alpha_2} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( \frac{\ln x_{\kappa_{\lambda}^-} w}{(-x_{\kappa_{\lambda}^-} w + x_{\kappa_{\eta}^-} w)(-x_{\kappa_{\lambda}^-} w + x_{\tilde{U}_{\alpha}^i} w)(-x_{\kappa_{\lambda}^-} w + x_{\tilde{U}_l^I} w)} \right. \\
& \left. - \frac{x_{iw} \ln x_{\kappa_{\lambda}^-} w}{(-x_{\kappa_{\lambda}^-} w + x_{\kappa_{\eta}^-} w)(-x_{\kappa_{\lambda}^-} w + x_{mw})(-x_{\kappa_{\lambda}^-} w + x_{\tilde{U}_{\alpha}^i} w)(-x_{\kappa_{\lambda}^-} w + x_{\tilde{U}_l^I} w)} \right) \\
& + \frac{16}{3} x_{\kappa_{\lambda}^-} w \sqrt{x_{\tilde{g}w} x_{iw}} \left( Z_{\tilde{U}^i}^{2\alpha_1} Z_{\tilde{U}^i}^{1\alpha_2} + Z_{\tilde{U}^i}^{1\alpha_1} Z_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_d Z_{\tilde{U}^i}^{1i} Z_-^{2\lambda}}{\sqrt{2}} \frac{h_b Z_{\tilde{U}^i}^{1\alpha_2} Z_-^{2\lambda}}{\sqrt{2}} \\
& \left( -Z_{\tilde{U}^j}^{1\beta} Z_+^{1\eta} + \frac{m_{uj} Z_{\tilde{U}^j}^{2\beta} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \left( -Z_{\tilde{U}^j}^{1\beta} Z_+^{1\eta} + \frac{m_{uj} Z_{\tilde{U}^j}^{2\beta} Z_+^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( \frac{\ln x_{\kappa_{\lambda}^-} w}{(-x_{\kappa_{\lambda}^-} w + x_{\kappa_{\eta}^-} w)(-x_{\kappa_{\lambda}^-} w + x_{\tilde{U}_{\alpha}^i} w)(-x_{\kappa_{\lambda}^-} w + x_{\tilde{U}_l^I} w)} \right. \\
& \left. - \frac{x_{iw} \ln x_{\kappa_{\lambda}^-} w}{(-x_{\kappa_{\lambda}^-} w + x_{\kappa_{\eta}^-} w)(-x_{\kappa_{\lambda}^-} w + x_{iw})(-x_{\kappa_{\lambda}^-} w + x_{\tilde{U}_{\alpha}^i} w)(-x_{\kappa_{\lambda}^-} w + x_{\tilde{U}_l^I} w)} \right) \\
& + \frac{16}{3} x_{\kappa_{\eta}^-} w \sqrt{x_{\tilde{g}w} x_{iw}} \left( Z_{\tilde{U}^m}^{2i} Z_{\tilde{U}^i}^{1\alpha_2} + Z_{\tilde{U}^i}^{1\alpha_1} Z_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_b Z_{\tilde{U}^j}^{1\beta} Z_-^{2\eta}}{\sqrt{2}} \frac{h_d Z_{\tilde{U}^j}^{1\beta} Z_-^{2\eta}}{\sqrt{2}} \\
& \left( -Z_{\tilde{U}^i}^{1\alpha_1} Z_+^{1\lambda} + \frac{m_{ui} Z_{\tilde{U}^i}^{1\alpha_1} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \left( -Z_{\tilde{U}^i}^{1\alpha_2} Z_+^{1\lambda} + \frac{m_{ui} Z_{\tilde{U}^i}^{2\alpha_2} Z_+^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( \frac{\ln x_{\kappa_{\eta}^-} w}{(x_{\kappa_{\lambda}^-} w - x_{\kappa_{\eta}^-} w)(-x_{\kappa_{\eta}^-} w + x_{\tilde{U}_{\alpha}^i} w)(-x_{\kappa_{\eta}^-} w + x_{\tilde{U}_l^I} w)} \right. \\
& \left. - \frac{x_{iw} \ln x_{\kappa_{\eta}^-} w}{(x_{\kappa_{\lambda}^-} w - x_{\kappa_{\eta}^-} w)(-x_{\kappa_{\eta}^-} w + x_{mw})(-x_{\kappa_{\eta}^-} w + x_{\tilde{U}_{\alpha}^i} w)(-x_{\kappa_{\eta}^-} w + x_{\tilde{U}_l^I} w)} \right)
\end{aligned}$$



$$\begin{aligned}
& \left( \frac{\ln x_{\tilde{U}_\alpha^i \text{w}}}{(x_{\kappa_\lambda^- \text{w}} - x_{\tilde{U}_\alpha^i \text{w}})(x_{\kappa_\eta^- \text{w}} - x_{\tilde{U}_\alpha^i \text{w}})(-x_{\tilde{U}_\alpha^i \text{w}} + x_{\tilde{U}_l^I \text{w}})} \right. \\
& \quad \left. - \frac{x_{i\text{w}} \ln x_{\tilde{U}_\alpha^i \text{w}}}{(x_{\kappa_\lambda^- \text{w}} - x_{\tilde{U}_\alpha^i \text{w}})(x_{\kappa_\eta^- \text{w}} - x_{\tilde{U}_\alpha^i \text{w}})(x_{i\text{w}} - x_{\tilde{U}_\alpha^i \text{w}})(-x_{\tilde{U}_\alpha^i \text{w}} + x_{\tilde{U}_l^I \text{w}})} \right) \\
& + \frac{16}{3} \sqrt{x_{\tilde{g}\text{w}} x_{i\text{w}}} x_{\tilde{U}_l^I \text{w}} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_b \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \\
& \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_+^{1\lambda} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_+^{2\lambda}}{\sqrt{2} m_{\text{w}} \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_+^{1\lambda} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \mathcal{Z}_+^{2\lambda}}{\sqrt{2} m_{\text{w}} \sin \beta} \right) \\
& \left( \frac{\ln x_{\tilde{U}_l^I \text{w}}}{(x_{\kappa_\lambda^- \text{w}} - x_{\tilde{U}_l^I \text{w}})(x_{\kappa_\eta^- \text{w}} - x_{\tilde{U}_l^I \text{w}})(x_{\tilde{U}_\alpha^i \text{w}} - x_{\tilde{U}_l^I \text{w}})} \right. \\
& \quad \left. - \frac{x_{i\text{w}} \ln x_{\tilde{U}_l^I \text{w}}}{(x_{\kappa_\lambda^- \text{w}} - x_{\tilde{U}_l^I \text{w}})(x_{\kappa_\eta^- \text{w}} - x_{\tilde{U}_l^I \text{w}})(x_{i\text{w}} - x_{\tilde{U}_l^I \text{w}})(x_{\tilde{U}_\alpha^i \text{w}} - x_{\tilde{U}_l^I \text{w}})} \right) \\
& + \frac{16}{3} \sqrt{x_{\tilde{g}\text{w}} x_{i\text{w}}} x_{\tilde{U}_l^I \text{w}} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_-^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_-^{2\lambda}}{\sqrt{2}} \\
& \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_{\text{w}} \sin \beta} \right) \left( -\mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_+^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^j}^{2\beta} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_{\text{w}} \sin \beta} \right) \\
& \left( \frac{\ln x_{\tilde{U}_l^I \text{w}}}{(x_{\kappa_\lambda^- \text{w}} - x_{\tilde{U}_l^I \text{w}})(x_{\kappa_\eta^- \text{w}} - x_{\tilde{U}_l^I \text{w}})(x_{\tilde{U}_\alpha^i \text{w}} - x_{\tilde{U}_l^I \text{w}})} \right. \\
& \quad \left. - \frac{x_{i\text{w}} \ln x_{\tilde{U}_l^I \text{w}}}{(x_{\kappa_\lambda^- \text{w}} - x_{\tilde{U}_l^I \text{w}})(x_{\kappa_\eta^- \text{w}} - x_{\tilde{U}_l^I \text{w}})(x_{i\text{w}} - x_{\tilde{U}_l^I \text{w}})(x_{\tilde{U}_\alpha^i \text{w}} - x_{\tilde{U}_l^I \text{w}})} \right) + (i \leftrightarrow j) , \tag{119}
\end{aligned}$$

$$\phi_3^{p\tilde{g}} = -2\phi_2^{p\tilde{g}} , \tag{120}$$

$$\begin{aligned}
\phi_4^{p\tilde{g}} = & -\frac{16}{3} \sqrt{x_{\kappa_\lambda^- \text{w}} x_{g\text{w}}} \mathcal{Z}_{\tilde{D}^3}^{1n} \mathcal{Z}_{\tilde{D}^1}^{2m} \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2\lambda}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_-^{1\lambda} + \frac{h_b \mathcal{Z}_{\tilde{D}^3}^{2m} \mathcal{Z}_-^{2\lambda}}{\sqrt{2}} \right) \\
& \left( -\mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2n} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_{\text{w}} \sin \beta} \right) \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c}) (x_{\kappa_\lambda^- \text{w}}, x_{\tilde{U}_\alpha^i \text{w}}, x_{\kappa_\eta^- \text{w}}, x_{j\text{w}}, x_{g\text{w}}, x_{\tilde{D}_\gamma^3 \text{w}}, x_{\tilde{D}_\delta^1 \text{w}}) \\
& -\frac{32}{3} \sqrt{x_{g\text{w}} x_{j\text{w}}} \mathcal{Z}_{\tilde{D}^3}^{1n} \mathcal{Z}_{\tilde{D}^1}^{2m} \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_-^{2\lambda}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_-^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2n} \mathcal{Z}_-^{2\eta}}{\sqrt{2}} \right) \\
& \frac{m_{u^j} \mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_+^{2\lambda}}{\sqrt{2} m_{\text{w}} \sin \beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_+^{1\eta} + \frac{m_{u^j} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_+^{2\eta}}{\sqrt{2} m_{\text{w}} \sin \beta} \right) \\
& F_A^{1a} (x_{\kappa_\lambda^- \text{w}}, x_{\tilde{U}_\alpha^i \text{w}}, x_{\kappa_\eta^- \text{w}}, x_{j\text{w}}, x_{g\text{w}}, x_{\tilde{D}_\gamma^3 \text{w}}, x_{\tilde{D}_\delta^1 \text{w}})
\end{aligned}$$

$$\begin{aligned}
& -\frac{32}{3} \sqrt{x_{\kappa_{\lambda}^-} x_{\kappa_{\eta}^-} x_{gw} x_{jw}} \mathcal{Z}_{\tilde{D}^3}^{1n} \mathcal{Z}_{\tilde{D}^1}^{2m} \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{-}^{1\lambda} + \frac{h_b \mathcal{Z}_{\tilde{D}^3}^{2m} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \right) \\
& \frac{m_{uj} \mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_A^0(x_{\kappa_{\lambda}^-}, x_{\tilde{U}_{\alpha}^i}, x_{\kappa_{\eta}^-}, x_{jw}, x_{gw}, x_{\tilde{D}_{\gamma}^3}, x_{\tilde{D}_{\delta}^1}) \\
& -\frac{16}{3} \sqrt{x_{\kappa_{\eta}^-} x_{gw}} \mathcal{Z}_{\tilde{D}^3}^{1n} \mathcal{Z}_{\tilde{D}^1}^{2m} \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{m_{uj} \mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \\
& \frac{m_{uj} \mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\eta} + \frac{m_{uj} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \right) \\
& \left( F_A^{1a} - F_A^{1b} + F_A^{1c} \right) (x_{\kappa_{\lambda}^-}, x_{\tilde{U}_{\alpha}^i}, x_{\kappa_{\eta}^-}, x_{jw}, x_{gw}, x_{\tilde{D}_{\gamma}^3}, x_{\tilde{D}_{\delta}^1}) + (i \leftrightarrow j) , \tag{121}
\end{aligned}$$

$$\phi_5^{p\tilde{g}} = \frac{1}{4} \phi_4^{p\tilde{g}} , \tag{122}$$

$$\begin{aligned}
\phi_6^{p\tilde{g}} &= \frac{16}{3} \sqrt{x_{\kappa_{\lambda}^-} x_{\kappa_{\eta}^-}} \mathcal{Z}_{\tilde{D}^3}^{2n} \mathcal{Z}_{\tilde{D}^1}^{2m} \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{-}^{1\lambda} + \frac{h_b \mathcal{Z}_{\tilde{D}^3}^{2m} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \right) \\
& \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{-}^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2n} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \right) \\
& \left( F_A^{1a} - F_A^{1b} - F_A^{1c} \right) (x_{\kappa_{\lambda}^-}, x_{\tilde{U}_{\alpha}^i}, x_{\kappa_{\eta}^-}, x_{jw}, x_{gw}, x_{\tilde{D}_{\gamma}^3}, x_{\tilde{D}_{\delta}^1}) \\
& -\frac{64}{3} \sqrt{x_{gw} x_{iw}} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \\
& F_C^{1a}(x_{\tilde{U}_{\alpha_2}^i}, x_{\kappa_{\lambda}^-}, x_{\kappa_{\eta}^-}, x_{\tilde{U}_{\alpha_1}^i}, x_{iw}, x_{\tilde{U}_{\alpha}^i}, x_{gw}) \\
& +\frac{16}{3} \sqrt{x_{\kappa_{\eta}^-} x_{jw}} \mathcal{Z}_{\tilde{D}^3}^{2n} \mathcal{Z}_{\tilde{D}^1}^{2m} \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \\
& \left( -\mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{-}^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2n} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \right) \frac{m_{uj} \mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \\
& \left( F_A^{1a} + F_A^{1b} - F_A^{1c} \right) (x_{\kappa_{\lambda}^-}, x_{\tilde{U}_{\alpha}^i}, x_{\kappa_{\eta}^-}, x_{jw}, x_{gw}, x_{\tilde{D}_{\gamma}^3}, x_{\tilde{D}_{\delta}^1}) \\
& +\frac{16}{3} \sqrt{x_{\kappa_{\lambda}^-} x_{jw}} \mathcal{Z}_{\tilde{D}^3}^{2n} \mathcal{Z}_{\tilde{D}^1}^{2m} \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{-}^{1\lambda} + \frac{h_b \mathcal{Z}_{\tilde{D}^3}^{2m} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \right) \\
& \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \frac{m_{uj} \mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \left( F_A^{1a} + F_A^{1b} \right. \\
& \left. - F_A^{1c} \right) (x_{\kappa_{\lambda}^-}, x_{\tilde{U}_{\alpha}^i}, x_{\kappa_{\eta}^-}, x_{jw}, x_{gw}, x_{\tilde{D}_{\gamma}^3}, x_{\tilde{D}_{\delta}^1}) \\
& -\frac{16}{3} \mathcal{Z}_{\tilde{D}^3}^{2n} \mathcal{Z}_{\tilde{D}^1}^{2m} \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1j} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \frac{m_{uj} \mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \frac{m_{uj} \mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta}
\end{aligned}$$

$$\begin{aligned}
& \left( F_A^{2b} + F_A^{2c} - F_A^{2d} - F_A^{2e} - 2F_A^{2f} \right) x_{\kappa_{\lambda}^- w}, x_{\tilde{U}_{\alpha}^i w}, x_{\kappa_{\eta}^- w}, x_{jw}, x_{gw}, x_{\tilde{D}_3^3 w}, x_{\tilde{D}_8^1 w} \\
& - 32x_{\kappa_{\lambda}^- w} \sqrt{x_{gw} x_{iw}} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \\
& \left( \frac{\ln x_{\kappa_{\lambda}^- w}}{(-x_{\kappa_{\lambda}^- w} + x_{\kappa_{\eta}^- w})(-x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\alpha_2}^i w})(-x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\alpha_1}^i w})} \right. \\
& \left. - \frac{x_{iw} \ln x_{\kappa_{\lambda}^- w}}{(-x_{\kappa_{\lambda}^- w} + x_{\kappa_{\eta}^- w})(-x_{\kappa_{\lambda}^- w} + x_{iw})(-x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\alpha_2}^i w})(-x_{\kappa_{\lambda}^- w} + x_{\tilde{U}_{\alpha_1}^i w})} \right) \\
& - 32x_{\kappa_{\eta}^- w} \sqrt{x_{gw} x_{iw}} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \\
& \left( \frac{\ln x_{\kappa_{\eta}^- w}}{(x_{\kappa_{\lambda}^- w} - x_{\kappa_{\eta}^- w})(-x_{\kappa_{\eta}^- w} + x_{\tilde{U}_{\alpha_2}^i w})(-x_{\kappa_{\eta}^- w} + x_{\tilde{U}_{\alpha_1}^i w})} \right. \\
& \left. - \frac{x_{iw} \ln x_{\kappa_{\eta}^- w}}{(x_{\kappa_{\lambda}^- w} - x_{\kappa_{\eta}^- w})(-x_{\kappa_{\eta}^- w} + x_{iw})(-x_{\kappa_{\eta}^- w} + x_{\tilde{U}_{\alpha_2}^i w})(-x_{\kappa_{\eta}^- w} + x_{\tilde{U}_{\alpha_1}^i w})} \right) \\
& + 32x_{iw}^2 \sqrt{x_{gw} x_{iw}} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \\
& \frac{\ln x_{iw}}{(x_{\kappa_{\lambda}^- w} - x_{iw})(x_{\kappa_{\eta}^- w} - x_{iw})(-x_{iw} + x_{\tilde{U}_{\alpha_2}^i w})(-x_{iw} + x_{\tilde{U}_{\alpha_1}^i w})} \\
& - 32\sqrt{x_{gw} x_{iw}} x_{\tilde{U}_{\alpha_2}^i w} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \\
& \left( \frac{\ln x_{\tilde{U}_{\alpha_2}^i w}}{(x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\alpha_2}^i w})(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha_2}^i w})(-x_{\tilde{U}_{\alpha_2}^i w} + x_{\tilde{U}_{\alpha_1}^i w})} \right. \\
& \left. - \frac{x_{iw} \ln x_{\tilde{U}_{\alpha_2}^i w}}{(x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\alpha_2}^i w})(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha_2}^i w})(x_{iw} - x_{\tilde{U}_{\alpha_2}^i w})(-x_{\tilde{U}_{\alpha_2}^i w} + x_{\tilde{U}_{\alpha_1}^i w})} \right) \\
& - 32\sqrt{x_{gw} x_{iw}} x_{\tilde{U}_{\alpha_1}^i w} \left( \mathcal{Z}_{\tilde{U}^i}^{2\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} + \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{\tilde{U}^i}^{2\alpha_2} \right) \frac{h_d \mathcal{Z}_{\tilde{U}^i}^{1\alpha_1} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha_2} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \frac{h_b \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \frac{h_d \mathcal{Z}_{\tilde{U}^j}^{1\beta} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \\
& \left( \frac{\ln x_{\tilde{U}_{\alpha_1}^i w}}{(x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\alpha_1}^i w})(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha_1}^i w})(x_{\tilde{U}_{\alpha_2}^i w} - x_{\tilde{U}_{\alpha_1}^i w})} \right. \\
& \left. - \frac{x_{iw} \ln x_{\tilde{U}_{\alpha_1}^i w}}{(x_{\kappa_{\lambda}^- w} - x_{\tilde{U}_{\alpha_1}^i w})(x_{\kappa_{\eta}^- w} - x_{\tilde{U}_{\alpha_1}^i w})(x_{iw} - x_{\tilde{U}_{\alpha_1}^i w})(x_{\tilde{U}_{\alpha_2}^i w} - x_{\tilde{U}_{\alpha_1}^i w})} \right) + (i \leftrightarrow j) , \tag{123}
\end{aligned}$$

$$\phi_7^{p\bar{g}} = -\frac{16}{3} \sqrt{x_{\kappa_{\eta}^- w} x_{gw}} \mathcal{Z}_{\tilde{D}^3}^{2n} \mathcal{Z}_{\tilde{D}^1}^{1m} \left( -\mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{-}^{1\lambda} + \frac{h_b \mathcal{Z}_{\tilde{D}^3}^{2m} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \right) \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}}$$

$$\begin{aligned}
& \left( -\mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{-}^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2n} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \right) \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{\kappa_{\lambda}^- w}, x_{\tilde{U}_{\alpha}^i w}, x_{\kappa_{\eta}^- w}, x_{jw}, x_{gw}, x_{\tilde{D}_{m w}^1}, x_{\tilde{D}_{\delta w}^1}) \\
& - \frac{32}{3} \sqrt{x_{\kappa_{\lambda}^- w} x_{\kappa_{\eta}^- w} x_{gw} x_{jw}} \mathcal{Z}_{\tilde{D}^3}^{2n} \mathcal{Z}_{\tilde{D}^1}^{1m} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \left( -\mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{-}^{1\eta} + \frac{h_d \mathcal{Z}_{\tilde{D}^1}^{2n} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \right) \\
& \frac{m_{uj} \mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \\
& F_A^0(x_{\kappa_{\lambda}^- w}, x_{\tilde{U}_{\alpha}^i w}, x_{\kappa_{\eta}^- w}, x_{jw}, x_{gw}, x_{\tilde{D}_{\gamma w}^3}, x_{\tilde{D}_{\delta w}^1}) \\
& - \frac{32}{3} \sqrt{x_{gw} x_{jw}} \mathcal{Z}_{\tilde{D}^3}^{2n} \mathcal{Z}_{\tilde{D}^1}^{1m} \left( -\mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{-}^{1\lambda} + \frac{h_b \mathcal{Z}_{\tilde{D}^3}^{2m} \mathcal{Z}_{-}^{2\lambda}}{\sqrt{2}} \right) \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \\
& \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\lambda} + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \right) \frac{m_{uj} \mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \\
& F_A^{1a}(x_{\kappa_{\lambda}^- w}, x_{\tilde{U}_{\alpha}^i w}, x_{\kappa_{\eta}^- w}, x_{jw}, x_{gw}, x_{\tilde{D}_{\gamma w}^3}, x_{\tilde{D}_{\delta w}^1}) \\
& - \frac{16}{3} \sqrt{x_{\kappa_{\lambda}^- w} x_{gw}} \mathcal{Z}_{\tilde{D}^3}^{2n} \mathcal{Z}_{\tilde{D}^1}^{1m} \frac{h_b \mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{-}^{2\eta}}{\sqrt{2}} \frac{m_{uj} \mathcal{Z}_{\tilde{D}^3}^{1m} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \\
& \left( -\mathcal{Z}_{\tilde{U}^i}^{1\alpha} \mathcal{Z}_{+}^{1\lambda} \right) + \frac{m_{u^i} \mathcal{Z}_{\tilde{U}^i}^{2\alpha} \mathcal{Z}_{+}^{2\lambda}}{\sqrt{2} m_w \sin \beta} \frac{m_{uj} \mathcal{Z}_{\tilde{D}^1}^{1n} \mathcal{Z}_{+}^{2\eta}}{\sqrt{2} m_w \sin \beta} \\
& (F_A^{1a} - F_A^{1b} + F_A^{1c})(x_{\kappa_{\lambda}^- w}, x_{\tilde{U}_{\alpha}^i w}, x_{\kappa_{\eta}^- w}, x_{jw}, x_{gw}, x_{\tilde{D}_{\gamma w}^3}, x_{\tilde{D}_{\delta w}^1}) + (i \leftrightarrow j) , \tag{124}
\end{aligned}$$

$$\phi_8^{p\bar{q}} = \frac{1}{4} \phi_7^{p\bar{q}} . \tag{125}$$

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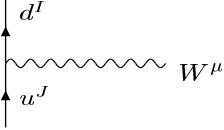
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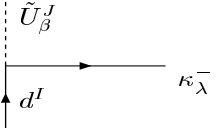
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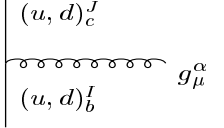
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(a)   $W^\mu \quad -\frac{ie}{\sqrt{2}\sin\theta_w}\gamma^\mu\omega_-C^{IJ}$

(b)   $H_k^- \quad \frac{ie}{\sqrt{2}\sin\theta_w}(h^{d^i}Z_H^{1k}\omega_- + \frac{m_{u,j}}{m_w\sin\beta}Z_H^{2k}\omega_+)C^{IJ}$

(c)   $\kappa_\lambda^- \quad \frac{ie}{\sin\theta}((-Z_{\tilde{U}^I}^{1\beta*}Z_+^{1\lambda} + \frac{m_{u,s}}{\sqrt{2}m_w\sin\beta}Z_{\tilde{U}^i}^{2\beta*})\omega_- + h^{d^I}Z_{\tilde{U}^I}^{1\beta*}Z_+^{2\lambda*}\omega_+)$

(d)   $g_\mu^\alpha \quad -ig_sT_{bc}^a\gamma_\mu\delta^{IJ}$

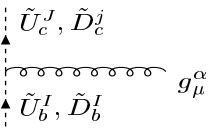
(e)   $g_\mu^\alpha \quad -ig_sT_{bc}^a(P+k)_\mu\delta^{IJ}$

Figure 1: The Feynman-rules which are adopted in the calculations (Part I).

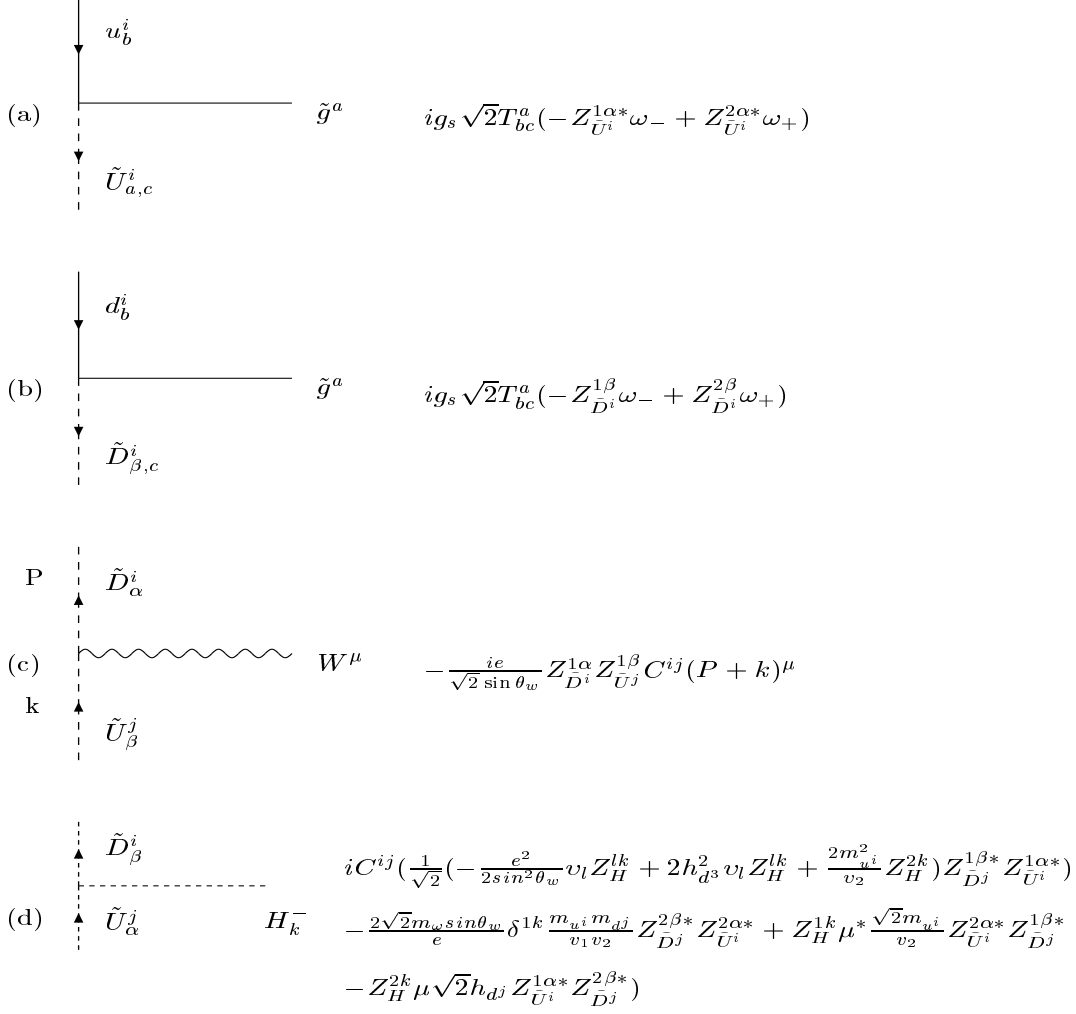


Figure 2: The Feynman-rules which are adopted in the calculations (Part II).

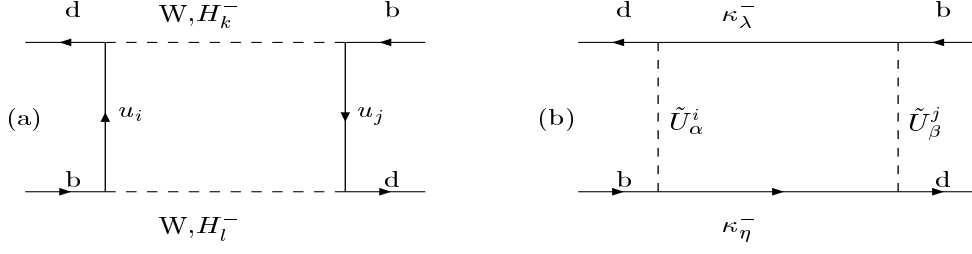


Figure 3: The box-diagrams contributing to the  $B^0 - \overline{B}^0$  mixing in the supersymmetric model with minimal flavor violation. In the calculations, the crossed diagrams should be included.

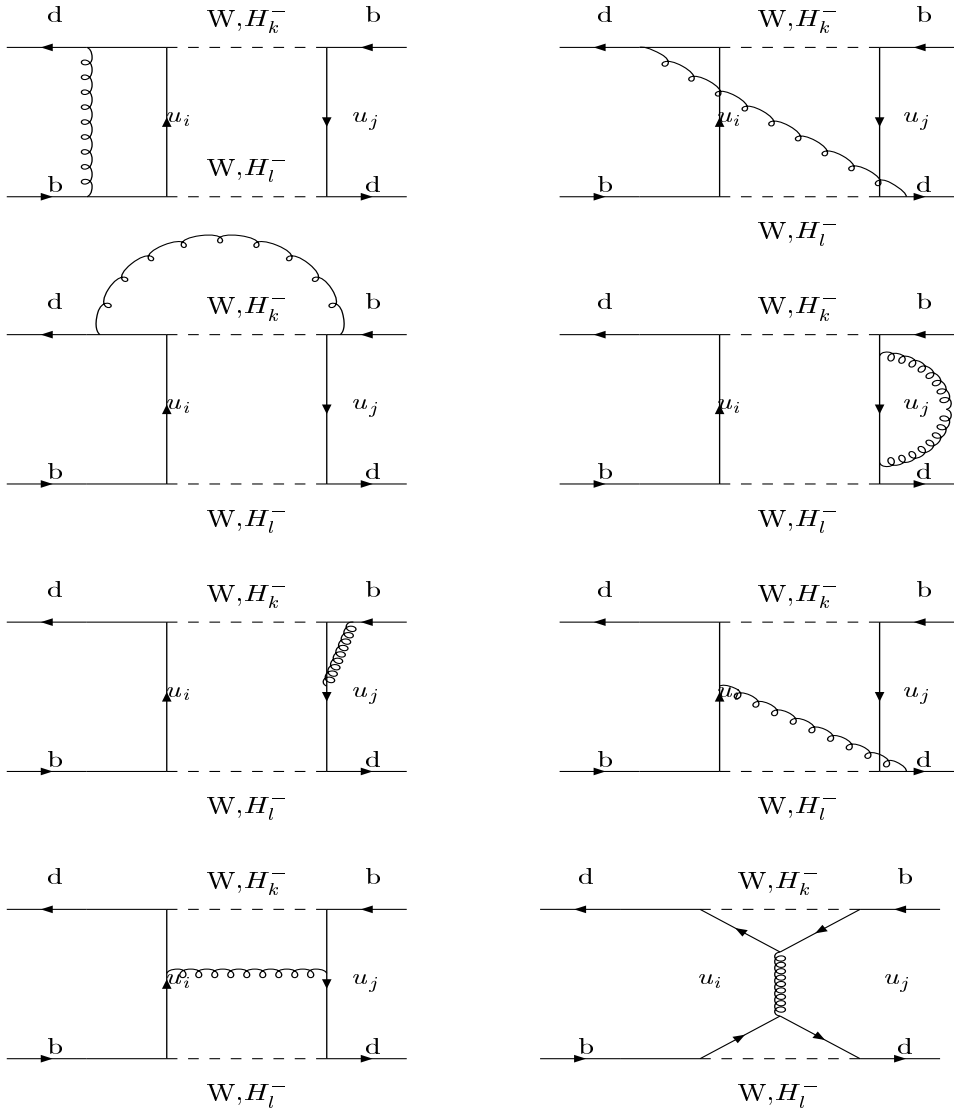


Figure 4: The diagrams responsible for QCD-corrections in the framework of the SM and THDM. In the calculations, the crossed diagrams should be included.

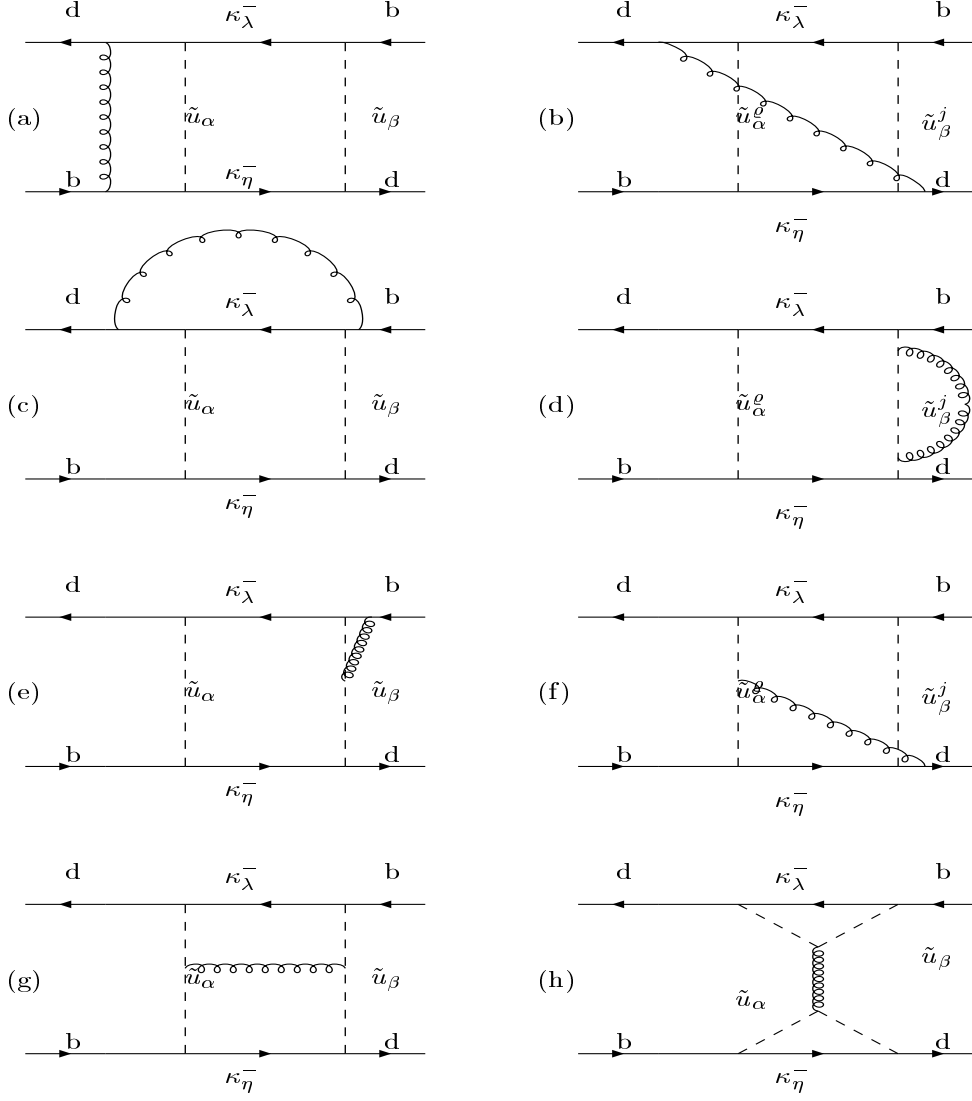


Figure 5: The diagrams responsible for QCD-corrections caused by the gluon sector of the supersymmetric model with minimal flavor violation. In the calculations, the crossed diagrams should be included.



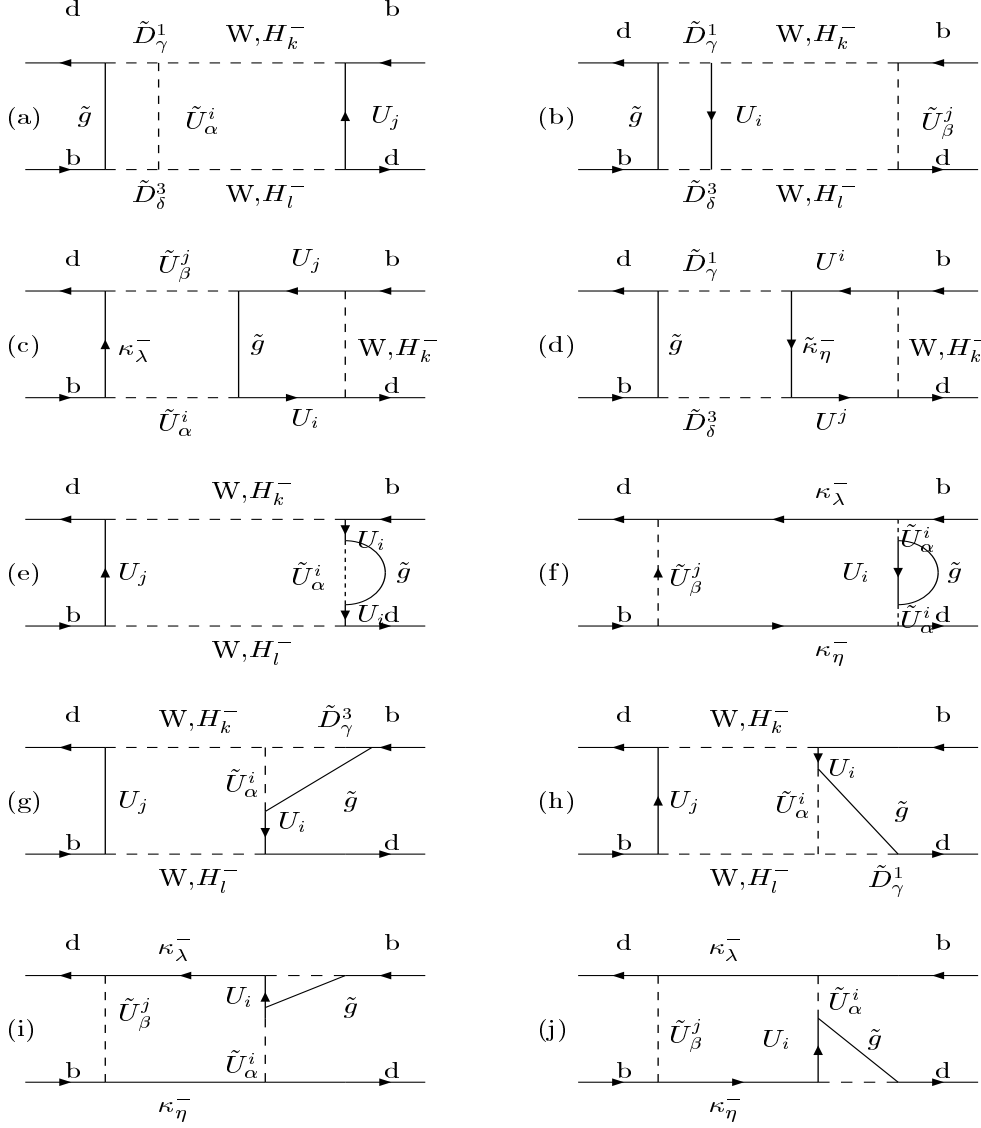


Figure 6: The diagrams responsible for QCD-corrections caused by the gluino sector of the supersymmetric theory with minimal flavor violation. In the calculations, the crossed diagrams should be included.

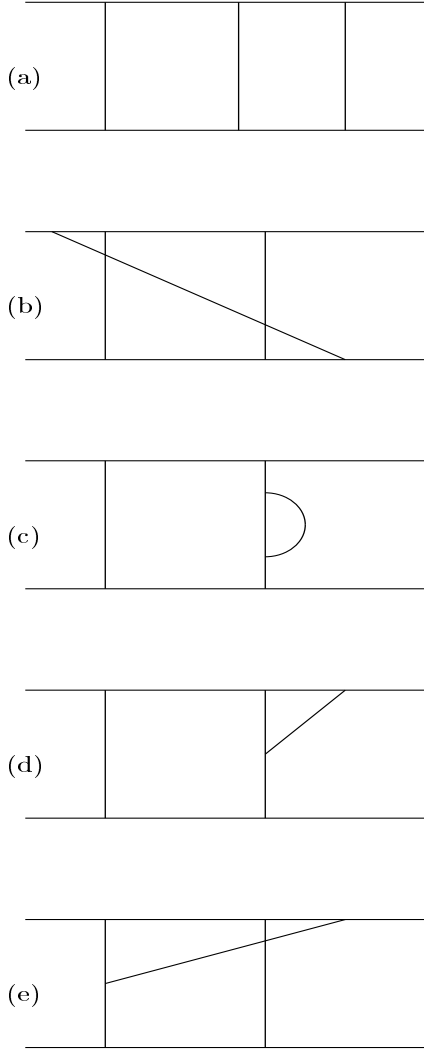


Figure 7: The five topological classes of diagrams appearing in the NLO-corrections to  $B^0 - \overline{B}^0$ .

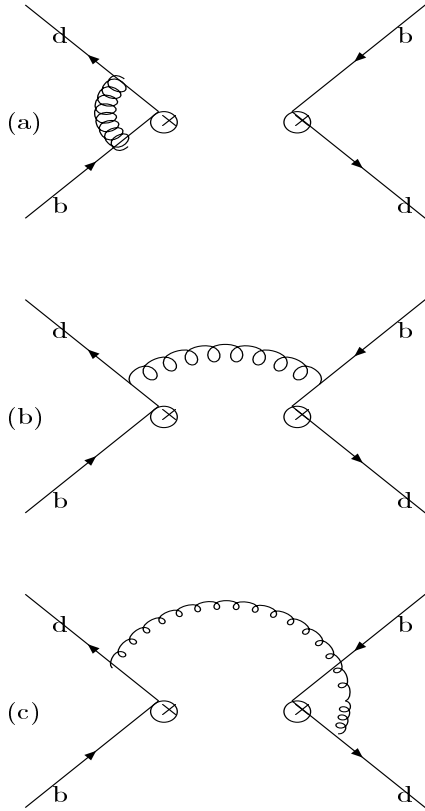


Figure 8: Classes of diagrams in the effective theory contributing to  $Q_i$  up to order  $\alpha_s$ .

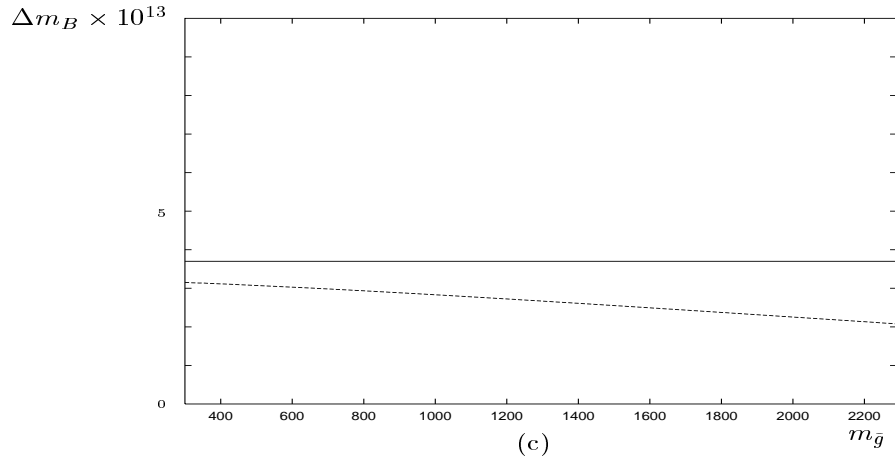
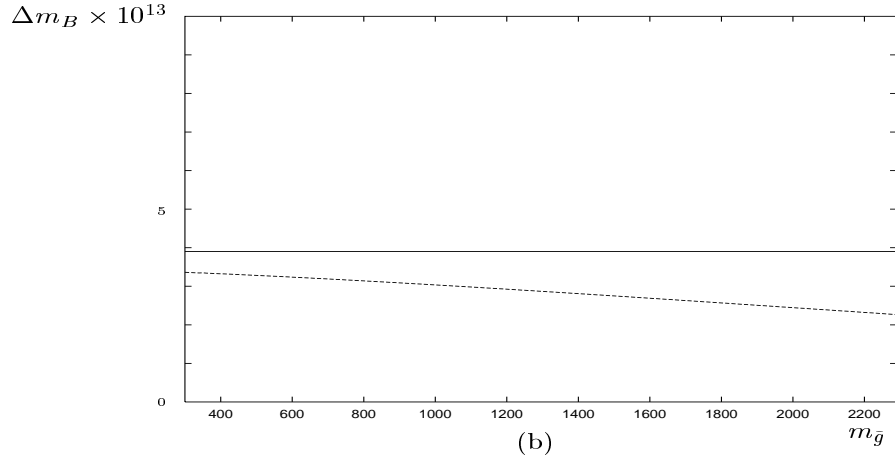
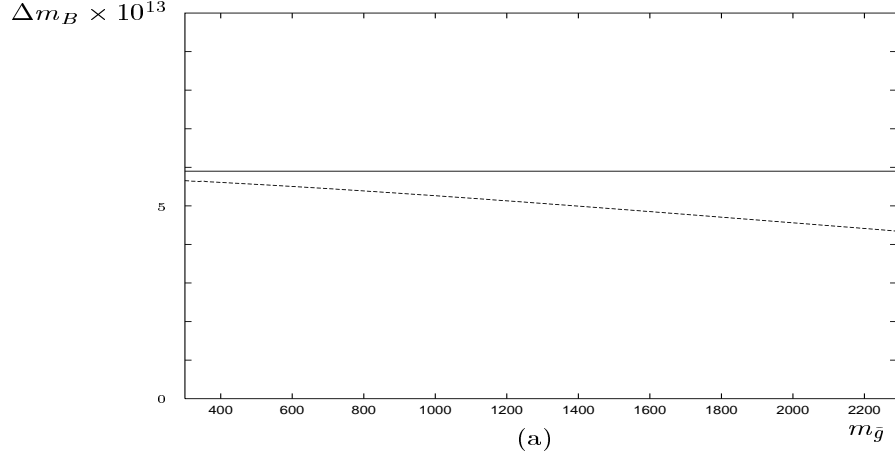


Figure 9: The  $\Delta m_B$  versus the gluino mass with  $\tan \xi_{\tilde{U}I} = \tan \zeta_{\tilde{B}} = \tan \zeta_{\tilde{D}} = 0$ . and (a)  $\tan \beta = 1$ , (b)  $\tan \beta = 5$ , (c)  $\tan \beta = 30$ . The dot-line corresponds to the results including the gluino-corrections and solid-line corresponds to that without the gluino-corrections. The other parameters are taken as in the text.

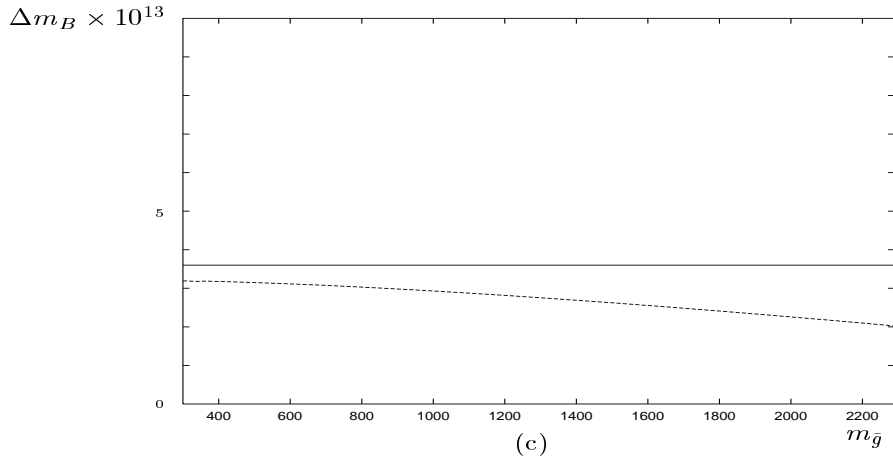
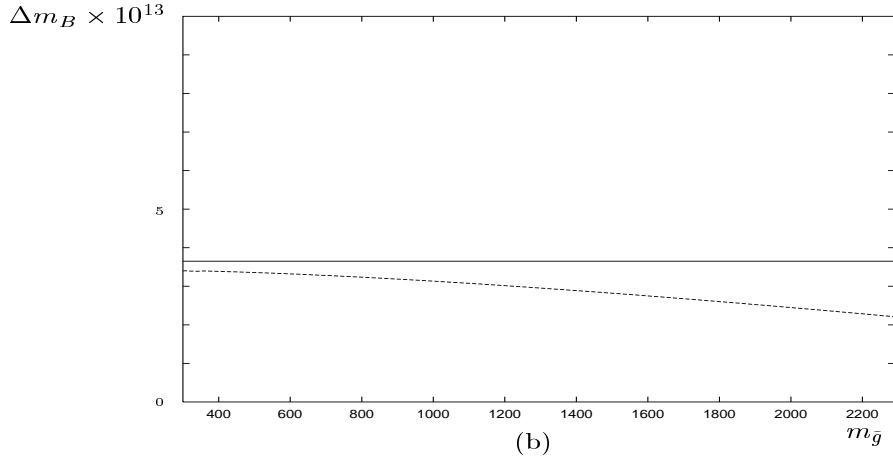
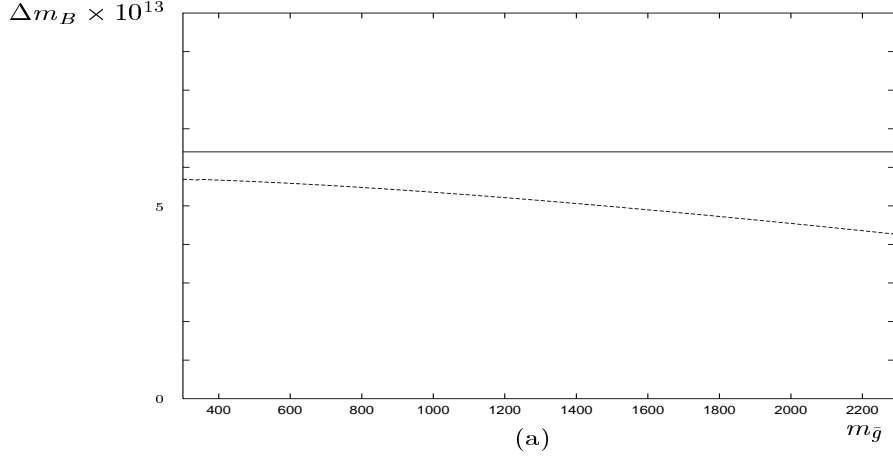


Figure 10: The  $\Delta m_B$  versus the gluino mass with  $\tan \xi_{\tilde{U}I} = \tan \zeta_{\tilde{B}} = \tan \zeta_{\tilde{D}} = 0.1$ . and (a)  $\tan \beta = 1$ , (b)  $\tan \beta = 5$ , (c)  $\tan \beta = 30$ . The dot-line corresponds to the results including the gluino-corrections and solid-line corresponds to that without the gluino-corrections. The other parameters are taken as in the text.

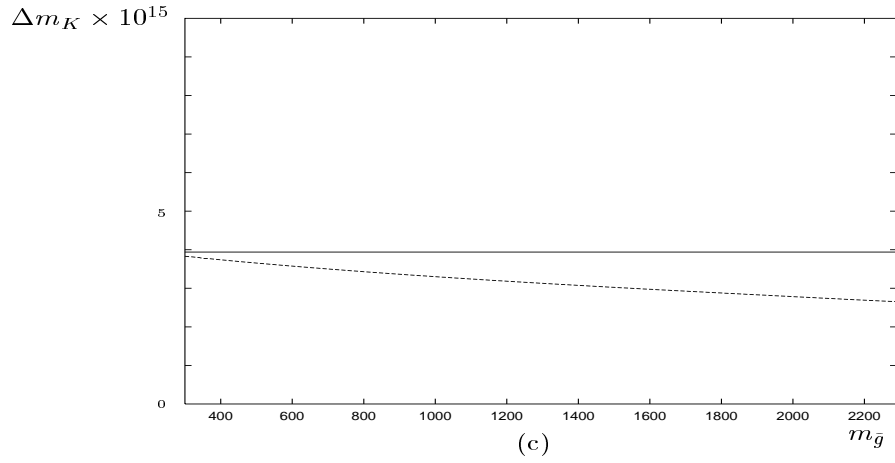
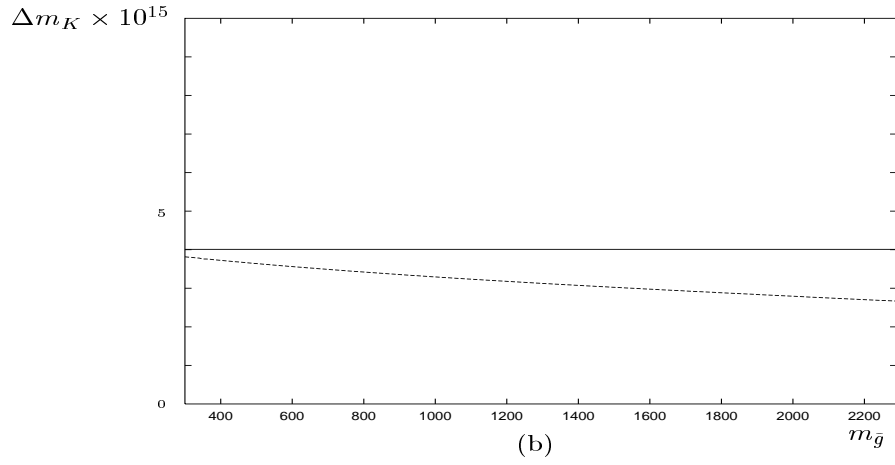
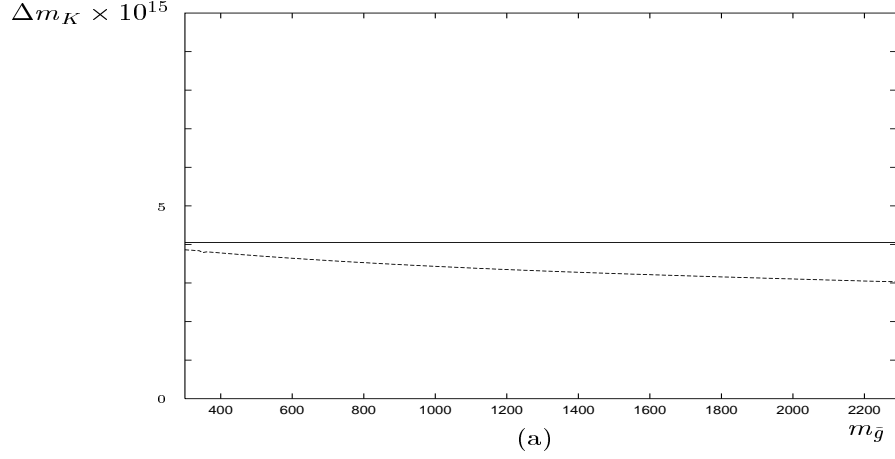


Figure 11: The  $\Delta m_K$  versus the gluino mass with (a)  $\tan \beta = 1.5$ , (b)  $\tan \beta = 5$ , (c)  $\tan \beta = 30$ , where  $\tan \xi_{\tilde{U}^I} = \tan \zeta_{\tilde{B}} = \tan \zeta_{\tilde{D}} = 0$ . The dot-line corresponds to the results including the gluino-corrections and solid-line corresponds to that without the gluino-corrections. The other parameters are taken as in the text.